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Abstract

We study the optimal design of a central bank digital currency (CBDC) in an environment where agents sort into cash, CBDC, and bank deposits according to their preferences over anonymity and security; and where network effects make the convenience of a payment instrument depend on the number of its users. A CBDC can be designed with attributes similar to cash or deposits, and can be interest bearing: a CBDC that closely competes with deposits depresses bank credit and output, while a cash-like CBDC may lead to the disappearance of cash. Then, the optimal CBDC design trades of bank intermediation against the social value of maintaining diverse payment instruments. When network effects matter, an interest-bearing CBDC alleviates the central bank's tradeoffs.

JEL Classification: E41, E58, G21

Keywords: CBDC, Fintech, digital currency, Financial Intermediation, network effects

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Designing Central Bank Digital Currencies*

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September 23, 2020

Abstract

We study the optimal design of a central bank digital currency (CBDC) in an environment where agents sort into cash, CBDC, and bank deposits according to their preferences over anonymity and security; and where network effects make the convenience of a payment instrument depend on the number of its users. A CBDC can be designed with attributes similar to cash or deposits, and can be interest bearing: a CBDC that closely competes with deposits depresses bank credit and output, while a cash-like CBDC may lead to the disappearance of cash. Then, the optimal CBDC design trades off bank intermediation against the social value of maintaining diverse payment instruments. When network effects matter, an interest-bearing CBDC alleviates the central bank's tradeoff.

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1 Introduction

Payment systems and, more fundamentally, money are evolving rapidly. Developments in digital networks and information technology and the increasing share of internet-based retailing have created the demand and technological space for digital transactions that have the potential to radically change payment and financial intermediation systems. Central banks have been pondering whether and how to adapt. Many are exploring the idea of issuing central bank digital currency (CBDC) - a new type of fiat money that expands digital access to central bank reserves to the public at large, instead of restricting it to commercial banks.[1] But would the CBDC resemble deposits by coming in the form of an account at the central bank, or would it come closer to cash, materializing as a digital token? How much anonymity could it offer users? Would it pay interest rates like a bank deposit, or would its nominal return be fixed at naught, like cash?[2] In this paper, we build a theoretical framework geared at analyzing the relation between the design of a CBDC, the demand for types of money, and financial intermediation.

Swings in the usage of payment instruments become particularly disruptive in the presence of network effects. For example, with a decline in the use of cash, banks may cut back on ATMs or shops may refuse to accept cash, a process currently underway in Sweden (Sveriges Riksbank, 2018). Because of such network effects, payment instruments may disappear when their use falls below a critical threshold, and the successful introduction of a CBDC could risk tipping the balance. Network effects are a critical feature of our model.

Our starting point is a (static) economy with banks, firms and households. In this economy, banks collect deposits, extend credit to firms, and create social value in doing

[1] In a survey of 66 central banks, over one-third of them perceived CBDC as a possibility in the medium term (Boar et al., 2020). Notably, the Swedish Riksbank and the People's Bank of China are expected to decide on introducing CBDCs for domestic retail payments in the near term, while the central banks of the Bahamas, Cambodia, China, the Eastern Caribbean, Mauritius, South Korea, Ukraine and Uruguay have run or are running pilots (Auer et al., 2020; Kiff et al., 2020).

[2] See Aymanns et al. (2020), Bank for International Settlements (2018) and Mancini-Griffoli et al. (2018) for other, operational design aspects of CBDCs, such as the means to disseminate and clear CBDCs.

so: firms' projects are worth less if they cannot receive bank loans.[3] Both banks and firms engage in perfect competition.

Households face a Hotelling linear city, where they aim to minimize the distance between the available forms of money and their preferences. In particular, households have heterogeneous preferences over anonymity and security in payments. We represent these preferences by an interval with cash and deposits at opposite ends: cash provides anonymity in transactions, while bank deposits are more secure.[4] A CBDC can take any point on this interval, depending on its design. For instance, a central bank could provide partial anonymity (e.g., towards third parties but not the authorities), set transaction limits below which anonymity is retained, or make anonymity conditional, only to be lifted by a court order - possibilities under consideration in central banks' studies of CBDCs (European Central Bank, 2019; Kiff et al., 2020; Mancini-Griffoli et al., 2018).

Overall, taking into account the design of the CBDC, households optimally sort into different types of money according to three considerations: their (heterogeneous) preferences, network effects deriving from the relation between the convenience of using a payment instrument and the number of its users, and the interest rates offered on deposits and possibly on the CBDC.

Our framework provides novel and policy-relevant insights for welfare analysis and optimal CBDC design. In our model, variety in payment instruments increases welfare because of the heterogeneity in household preferences. The CBDC then has social value due to its ability to blend features of cash and deposits. As emphasized by Lagarde (2018), there is a potential demand for partially anonymous means of payment that can, for example, protect consumers from the use of personal transactions data for credit assessments, a possibility that is increasingly enabled by technological developments.[5] Indeed, the ability of monop-

[3] We parameterize and vary the degree to which bank financing of firms provides efficiency gains. On the special role of depository institutions in intermediation, see Diamond and Rajan (2001) and Donaldson et al. (2018), as well as Merrouche and Nier (2012) for supporting empirical evidence.

[4] Empirical research on the choice of payment instruments attributes a central role to heterogeneous preferences (Shy, 2019; Wakamori and Welte, 2017).

[5] This forms the basis for the microfoundations that we develop in Appendix D.

olistic private payment providers to take advantage of data access is cited as a rationale for CBDC by the central banks of Canada, China, Norway, and Sweden.[6] Borgonovo et al. (2019) conduct an experiment to measure the value that households place on anonymity in payments, specifically deriving the demand for different potential designs of CBDC. They find that the degree of anonymity is an important determinant of potential CBDC demand.[7] More generally, various authors discuss the legitimate reasons for households' demand for privacy in payments (Garratt and van Oordt, 2019; Kahn et al., 2005; McAndrews, 2017).[8]

In spite of the social value of an intermediate payment instrument, introducing a CBDC has welfare costs to the extent that it crowds out demand for cash and deposits.[9] Specifically, a cash-like CBDC can reduce the demand for cash beyond the point where network effects cause the disappearance of cash. But a deposit-like CBDC causes an increase in deposit and loan rates, and a contraction in bank lending to firms. Because of relationship lending frictions, this decline in bank intermediation also curtails investment and output.[10]

We show that the welfare-optimal CBDC design hinges on whether the CBDC is interest bearing, and whether network effects matter. When the CBDC is not interest bearing, its similarity to cash becomes the sole design instrument. The more important the role of banks in alleviating lending frictions, the more cash-like the optimal CBDC design becomes. But network effects twist the optimal design problem, as the variety of payment instruments that

[6] See Kiff et al. (2020), and Table 2 (p.28) and discussions (p.20) in Mancini-Griffoli et al. (2018).

[7] For further discussions on experimental design to ascertain CBDC demand, see also Borgonovo et al. (2018) and Masciandaro (2018). In addition, Athey et al. (2017) experimentally test for the value of digital privacy in general (beyond payments). Moreover, using Canadian survey data, Huynh et al. (2020) find confirming evidence for the role of security features in the demand for money.

[8] Anonymous means of payment could also facilitate illicit activities, however (Rogoff, 2016; Wright et al., 2017). We consider this possibility in an extension in Appendix C.2.

[9] Nevertheless, a CBDC is certain to raise aggregate welfare in our framework, but only if it is optimally designed. Moreover, even when aggregate welfare rises, there are distributional effects, and some households are worse off due to CBDC availability. We analyze these distributional effects in Section 3.3, which also finds that a social planner that can observe household payment choices (but not their underlying preferences) cannot design a lump-sum transfer scheme to ensure that the introduction of a CBDC is a Pareto improvement.

[10] A central bank could attempt to mitigate the decline in bank lending by providing banks with cheap liquidity to replace lost deposits. However, this may not be feasible for two reasons. First, banks' ability to intermediate funds may depend on their reliance on deposits (see, e.g., Diamond and Rajan, 2001; Donaldson et al., 2018). Second, this policy would permanently expose the central bank to credit risk.

households value becomes challenging to sustain. A nonlinear optimal design pattern then emerges: increasing the value added of banks translates into a more cash-like optimal design up to a point, after which CBDC design is constrained to preserve cash. This is so unless bank-based intermediation is sufficiently precious that the CBDC is introduced with a design that eliminates cash, and replaces it with a less than fully anonymous CBDC.

As long as network effects do not constrain policy, the CBDC interest rate is best kept at zero, because it brings about price distortions in the households' choice of payment instruments. Unlike the interest rates on bank deposits, there is no production underlying the payment of interest on the CBDC, which is funded with a lump-sum tax. Hence, the CBDC interest rate is a suboptimal tool compared to the design of CBDC payment attributes, which optimally center on meeting some households' demand.

However, access to an adjustable CBDC interest rate makes a palpable difference to the central bank when network effects come to the fore. If the introduction of a CBDC threatens cash with extinction, a negative CBDC interest rate can compensate.[11] Indeed, when households care enough about payment instrument variety, an interest-bearing CBDC will optimally always keep cash alive, while limiting the CBDC's impact on bank intermediation. This is a finding of policy relevance, since all ongoing central bank CBDC initiatives center on CBDCs that do not bear interest.

In several extensions, we investigate whether considerations other than network effects can cause optimal CBDC rates to diverge from zero.[12] For instance, when banks have market power in lending, the central bank makes the CBDC compete harder with deposits, which leads optimal CBDC rates to diverge from zero, regardless of network effects. We also show that our results are robust to alternate production functions.

In another extension, we provide a model where households choose between cash and deposit-based payment services that are bundled with the provision of other services (e.g.,

[11] Beyond satisfying household preferences, the disappearance of cash may reduce economic activity when a portion of the population is unable or unwilling to transact with digital payment methods because of digital illiteracy or informality. See Chodorow-Reich et al. (2020) for an empirical assessment of such costs.

[12] See Appendix C.

credit provision related to transaction data). This microfounds a linear city of payment preferences, and highlights the potential demand for a CBDC whose attributes straddle those of existing payment systems.

Lastly, we consider an extension of our framework with a private digital currency and analyze the private issuer's design problem. The issuer cares about interest income as well as data acquisition, which depends positively on the size of the currency's user base and negatively on the degree of anonymity the currency offers. The privately optimal design of this currency and its interest rate are never welfare optimal, as the issuer fails to internalize externalities on money variety and bank intermediation.

Our paper is closely related to a recent and growing literature on CBDCs. One strand of this literature focuses on the impact of introducing a CBDC on the banking sector. Andolfatto (2018) and Chiu et al. (2019) develop models where a CBDC increases welfare by reducing banks' deposit market power, while a set of recent papers consider the relation between banking panics and the availability of a CBDC.[13] In Keister and Sanches (2019), a CBDC contributes to efficiency in exchange at the expense of crowding out deposits.

Compared to this literature, our first contribution is to highlight a tradeoff between preserving variety in payment instruments in the face of network effects, and mitigating the adverse effects of a CBDC on financial intermediation. Our second contribution is to show when and why this tradeoff is harder to overcome with a non interest-bearing CBDC than with a CBDC that offers an adjustable interest rate.[14]

Our paper also relates to the literature on payment systems as well as that on network effects. Our modeling of network effects follows closely on the seminal work of Katz and

[13]Namely, Böser and Gersbach (2020), Brunnermeier and Niepelt (2019), Fernández-Villaverde et al. (2020), and Skeie (2019). This issue also features prominently in policy discussions (Bech and Garratt, 2017; Cukierman, 2019; Kahn et al., 2019b).

[14]In our framework, CBDC interest rates represent any type of subsidy or cost associated with holding a CBDC. For example, the pilot conducted by the central bank of Uruguay offered subsidies to CBDC holders (Bergara and Ponce, 2018). Moreover, we focus on the steady-state effects of CBDC rates on financial intermediation and cash use, rather than their implications for monetary policy over the business cycle. On the relation between CBDC and monetary transmission, see Agarwal and Kimball (2015, 2019), Assenmacher and Krogstrup (2018), Barrdear and Kumhof (2016), Bordo and Levin (2017), Bjerg (2017), Davoodalhosseini (2018), Goodfriend (2016), Meaning et al. (2018), Niepelt (2020), and Parlour et al. (2020).

Shapiro (1985). While Katz and Shapiro (1985) study firms' decisions to introduce mutually compatible products, we focus on a social planner's decision to introduce and design a new payment instrument. In the literature on payment systems, the analysis and measurement of network effects centers on credit and debit card networks (Bounie et al., 2017; Chakravorti, 2010; Rochet and Tirole, 2006).[15] Instead, we study the impact of a new form of money on the demand for the existing payment instruments, and on welfare. Lastly, the value of variety in payment instruments has a similarity to the value of product variety in international trade (Krugman, 1979). However, our model does not build on an assumed love of variety, but rather on a heterogeneity in preferences that is best served by variety.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the optimal design of a CBDC. Section 4 concludes. Extensions of the model can be found in the appendices.[16]

2 A model of payment instruments

We consider a financial economy populated by households, banks, firms, and a central bank that aims to maximize welfare. Events unfold over two stages. In the first stage, the central bank decides whether and in what form to introduce a CBDC.[17] In the second stage, households choose between holding cash, bank deposits and (if introduced) CBDC for their transactions, and banks use deposits collected from households to extend loans to firms, which in turn produce a consumption good.

Along with the rate of interest offered (if any), households value two attributes of a payment instrument – anonymity and security – with heterogeneous preferences over their relative importance. At the core of our model lies a tension between these two attributes, because delinking transactions from personal identity leads to a loss of traceability that

[15] The role of strategic coordination and adoption equilibria has also been considered in the literature on cryptocurrencies (Biais et al., 2019, 2018; Bolt and Van Oordt, 2019).

[16] All algebraic calculations are conducted in *Mathematica*. The file is available on request.

[17] See Appendix E for an extension of the model with a privately issued digital currency, instead of a CBDC.

creates risks for the holder. For example, while depository accounts are relatively safe and traceable, cash is vulnerable to accidental loss and theft.[18] It is precisely the fact that cash can be lost without any claim that also makes it perfectly anonymous.

This intrinsic link between the degree of anonymity of a means of payment and the difficulty of keeping it safe extends to the realm of digital money.[19] A CBDC can perhaps only approach the anonymity of cash if it takes the form of a token that is accessible through user accounts that are not independently verified, or a nameless payment card that can be purchased at stores or online.[20] These forms of CBDC would also suffer from the risks of loss and theft associated with cash, either physically (e.g., card loss) or digitally (e.g., the untraceable loss of account information). At the other extreme, an account at the central bank that can be opened only using official identification would mimic the security and traceability of bank deposits.

Unlike cash and deposits, however, CBDC can be designed to blend intermediate amounts of anonymity and security.[21] For example, anonymity may be preserved vis-à-vis third parties only, and transactions can be recorded but not accessed by the central bank unless a transaction size limit is breached and/or there is suspicion of wrongdoing. Such possibilities are being considered in central banks' studies of CBDCs (European Central Bank, 2019; Kiff et al., 2020; Mancini-Griffoli et al., 2018).

Considering, for instance, the idea of a transaction size threshold below which the central bank promises not to access transaction records, highlights the possibility of "calibrating"

[18]We abstract from default risk on bank deposits, which is negligible in normal times due to deposit insurance and implicit bailout guarantees.

[19]See also Engert and Fung (2017) and Kahn et al. (2019a).

[20]It is an open question whether any digital currency could truly replicate the level of anonymity associated with cash. Cryptocurrencies, such as Bitcoin, are often characterized as pseudonymous, because tracking mechanisms may be possible (Goldfeder et al., 2018). In the case of a CBDC, a nameless payment card is one of the options under consideration in Sweden (Sveriges Riksbank, 2018). A related example is the Marshall Islands, where the new legal tender cryptocurrency (https://sov.foundation/) will be printable at ATMs and exchangeable like cash.

[21]While some legal jurisdictions allow for deposit accounts that offer a degree of anonymity, these accounts are typically incompatible with payment services. Moreover, providing anonymity in deposits may undermine their complementarity with relationship lending (see, e.g., Donaldson et al., 2018).

anonymity, and its inherent link to security.[22] The level of the threshold would effectively determine how much anonymity (i.e., what fraction of transactions) is being offered on the CBDC. At the same time, households' ability to seek recourse when something goes wrong with a transaction below the threshold may be limited if the validity of the claim cannot be verified by the central bank without violating the promised anonymity (which the other party in the payment may not agree to forgo). Hence, security declines as anonymity rises.

We formalize these considerations in an anonymity-security scale $[0, 1]$, where higher values denote a greater extent of anonymity and, equivalently, a lesser degree of security. If we let x_j denote the place of each money type j in the anonymity-security scale, cash (denoted by c) is placed at the top of the scale $x_c = 1$, deposits (denoted by d) are at the bottom $x_d = 0$, and CBDC is placed at $x_{cbdc} = \theta$, where $\theta \in [0, 1]$ is a design parameter determined by the central bank. In addition, the central bank determines the (net) interest rate offered on the CBDC, r_{cbdc}, which we allow to take any (positive, zero or negative) value. The combination (θ, r_{cbdc}) thus describes CBDC design in our framework.

There are two important frictions in the model economy: relationship lending frictions and network externalities in payment transactions. Relationship lending frictions take the form of information asymmetries that bar households from lending directly to firms, which are endowed with positive net present value projects that require financing. Firms may then either rely on intermediation by banks or liquidate their projects. The importance of bank intermediation is proportionate to the gap between firm productivity A and the liquidation value of firm projects ϕ. When $(A - \phi)$ is higher, a given decline in bank deposits and credit leads to a sharper reduction in output and consumption.

Network externalities give rise to a disutility cost of relying on a payment instrument that is not commonly used. We denote by η_j the disutility cost of using money type j and

[22] In this case, public trust in the central bank would play a key role in determining whether the CBDC is actually seen as partly anonymous.

adopt the functional form

$$\eta_j = \max\{0, g(s_j)\} \;\; \forall j \in \{c, cbdc, d\} \tag{1}$$

$$g(1) < 0 \; , \; g(0) > 0 \; , \; g'(.) < -1$$

where s_j stands for the share of households holding a money type j. The functional form for η_j is restrictive in two ways. First, we implicitly assume that, for a given share of users s_j, network externalities are equivalent across different money types. Second, the maximum operator and the restrictions on $g(.)$ imply that network externalities only take hold when the share of households using a money type falls short of a threshold. We find it convenient to define this threshold as $\underline{s} \equiv g^{-1}(0)$, where $0 < \underline{s} < 1$. Once network externalities come into play, the restriction $g'(.) < -1$ ensures that they lead to a cascade: each household that switches to a different money type incentivizes another to switch until that type of money falls out of use. This setup helps preserve tractability by eliminating unstable equilibria and ensuring that the equilibrium shares of money types that remain in use are not affected by network externalities.

Below, we explain the activities of the households, banks, firms and the central bank in further detail. We then proceed to characterize the equilibria and conduct welfare analysis.

2.1 Agents and their optimal strategies

2.1.1 Household preferences

There is a unit continuum of households with preferences $h \in [0, 1]$ uniformly distributed over the anonymity-security scale.[23] Households start with identical endowments in terms of (atomistic) shares in firms and liquid funds, which are normalized to 1. Liquid funds are stored in a money type $j \in \{c, d, cbdc\}$ and used to purchase the consumption good at the

[23] We adopt a uniform distribution for the sake of tractability. Our qualitative results generalize to any single peaked distribution with continuous support and sufficient weight in the tails to ensure that, absent a CBDC, both deposits and cash are sustained as payment instruments.

end of the stage.[24]

Households cannot attain their preference h by mixing different forms of money in their transactions because a transaction is only as anonymous as the least anonymous payment instrument used. In other words, anonymity is undiversifiable. This notion is further explored in Appendix D, which provides an example of how a linear-city setup, in the manner of Hotelling (1929), can be microfounded for the case of payment preferences.[25] In that Appendix, the heterogeneity of payment preferences emanates from heterogeneity of consumption preferences, with households differing in what fraction of income they want to use for purchases of "sin goods" (e.g., alcohol, tobacco). Such goods lower their creditworthiness when purchases are observed by the deposit-taking institution, which also provides credit to households, and observes their incomes. This relates closely to, for example, the payment system in China, where most deposit-based transfers are processed by large Fintech providers, which are also major providers of credit. Households are shown to sort into different groups: those that want to consume few sin goods sort into deposits, and those with stronger preferences for sin goods sort into cash.[26] Appendix D therefore provides an illustration of heterogeneous preferences for payment data privacy.

2.1.2 Household optimization

The household budget constraint is given by

$$C_j = 1 + r_j - T + \pi \tag{2}$$

[24] We assume that all forms of money are traded at par.

[25] Spatial differentiation frameworks such as Hotelling (1929) or Salop (1979) have been used extensively to model agents' heterogeneous preferences (or other features) in settings ranging from the market for cereals (Schmalensee, 1978), to voting preferences (Stokes, 1963), to the degree of information asymmetry in banking (Hauswald and Marquez, 2006).

[26] As their incomes are observed, the households understand that the deposit-taking institution can infer that any part of their income not saved or used for consumption of other goods, is used to consume sin goods with cash. Therefore, the only way to effectively hide from the revelation of sin good consumption, is to opt out from the relationship with the deposit-taking institution altogether.

where C_j is consumption, π represents dividends from firm profits, r_j is the (net) interest earned on money holdings, so that

$$r_j = \begin{cases} 0 & \text{if } j = c \\ r_{cbdc} & \text{if } j = cbdc \\ r_d & \text{if } \ j = d \end{cases} \tag{3}$$

and $T = r_{cbdc} s_{cbdc}$ is a lump-sum tax used to fund interest rates on the CBDC (or equivalently a lump-sum transfer to redistribute revenues when $r_{cbdc} < 0$).[27]

The households' utility maximization problem can then be written as

$$\max_{j \in \{c,d,cbdc\}} U_h(j) = \rho C_j - |x_j - h| - \eta_j \tag{4}$$

subject to the budget constraint (2). Here, $|x_j - h|$ represents the utility cost of selecting a payment method that differs from the household's anonymity-security preference h while $\rho > 0$ denotes the marginal utility of consumption relative to payment preferences.[28] Network effects are captured by η_j, as defined by (1). The solution to the household's problem yields the following cut-off conditions for a household with preferences h to choose

$$\text{cash over CBDC} \quad : \quad 1 - h + \eta_c < |\theta - h| - \rho r_{cbdc} + \eta_{cbdc} \tag{5}$$

$$\text{cash over deposits} \quad : \quad 1 - h + \eta_c < h - \rho r_d + \eta_d \tag{6}$$

$$\text{CBDC over deposits} \quad : \quad |\theta - h| - \rho r_{cbdc} + \eta_{cbdc} < h - \rho r_d + \eta_d \tag{7}$$

[27] This can be interpreted as a zero-capital central bank: any revenue that the central bank makes is immediately paid out to households, and any capital shortfall leads directly to a recapitalization through a lump-sum tax.

[28] The manner in which we combine consumption with payment preferences bears similarity to the utility function adopted in Gopinath and Stein (2018).

2.1.3 Banks

Banks collect deposits d from households at net deposit rate r_d and extend loans l to firms at net loan rate R, with the budget constraint

$$l = d \tag{8}$$

The representative bank is risk neutral and a price taker in both deposit and loan markets, so that its profit maximization problem yields the first-order condition

$$r_d = R \tag{9}$$

which simply equates interest rates on deposits and loans.[29]

2.1.4 Firms

Firms are perfectly competitive and begin with an identical endowment of productive projects k_0 which require financing to implement.[30] The representative firm uses bank loans l to finance a portion $k = l$ of projects. Once implemented, these projects yield a payoff in terms of consumption goods with the technology

$$Y = \left(A - \frac{k}{2}\right) k \tag{10}$$

[29] See Appendix C.3 for an extension where we allow for market power in the bank loan market.

[30] We impose the restriction $k_0 > 1$ to ensure that lending frictions always bind such that $k < k_0$.

where $A > 1$ denotes productivity.[31] The remaining projects $(k_0 - k)$ are liquidated at a constant rate of return $\phi \in (0, 1)$.[32]

The representative firm's profit maximization problem can then be written as

$$\max_{k,l} \; Y + \phi \left(k^0 - k\right) - (1 + R)\, l \tag{11}$$

subject to (10) and $k = l$. This yields the first-order condition

$$1 + R = A - \phi - l \tag{12}$$

which can be interpreted as a downward sloping loan demand curve. Notably, a decline in the liquidation value ϕ raises firm demand for loans, reflecting the increased importance of bank intermediation. More generally, we can refer to $(A - \phi)$ as 'the value added from bank intermediation,' since it captures the value generated by channeling funds to firms with productivity A, instead of having the firms sell off their projects at value ϕ. One can think of this from a cross-country perspective, where some countries are more reliant on bank-based intermediation and others less so, since in some countries relationship lending frictions are easier to overcome by alternate (i.e., nonbank) means.

[31] We adopt a quadratic functional form in the interest of tractability. Appendix C.1 considers a constant returns to scale technology as an alternative. In a derivation available upon request, we also generalize the quadratic technology to the form $Y = \left(A - \Gamma \frac{k}{2}\right) k$ and show that results are robust to varying Γ.

[32] The liquidation value is also in terms of consumption goods. The liquidation of projects can be microfounded in a framework similar to Stein (2012) where projects are sold to outside buyers with a lower marginal valuation. While we do not explicitly incorporate outside buyers into our model, doing so would have no impact on welfare provided these buyers are non-residents and/or projects are priced at their opportunity cost to outside buyers. In the interest of tractability, we also assume that funds from liquidated projects cannot be used for financing other projects. This could be due to a combination of information asymmetries and timing. For example, the time required for outside buyers to verify and pay for a project may exhaust the time for implementation by firms.

2.1.5 The central bank

The central bank aims to maximize social welfare, defined as the sum of household utilities

$$W = \int_{h\in[0,1]} U_h\left(j^*\left(h\right)\right) dh \tag{13}$$

where $j^*\left(h\right)$ denotes the payment instrument selected by household h. In doing so, the central bank decides whether to introduce a CBDC, and if introduced, its design characteristics (θ, r_{cbdc}).[33] If a CBDC is introduced, the central bank's design problem is given by

$$\max_{\theta\in[0,1], r_{cbdc}} \int_{h\in[0,1]} U_h\left(j^*\left(h\right)\right) dh \tag{14}$$

subject to a design constraint

$$s_{cbdc} \geq \underline{s} \tag{15}$$

which ensures that there is sufficient uptake of the CBDC to overcome network effects.[34]

In our framework, the central bank only considers the introduction of a single CBDC. We acknowledge that this is a potential limitation of our approach, because there might be cases where the introduction of multiple CBDCs would be socially optimal in the context of the model.

Nevertheless, the presence of network effects would limit the number of CBDCs that can be introduced, which would imply that the key tradeoff that the central bank faces, providing households with more variety of payment instruments while limiting the impact

[33] An implicit assumption in our model is that the central bank does not allow any agent to take a short position in CBDC (i.e., the central bank does not grant CBDC credit to other parties). This precludes arbitrage opportunities by entities without payment preferences, such as banks, which might prefer funding themselves with CBDC rather than deposits. Based on CBDC studies currently underway at central banks, we consider this a realistic assumption.

[34] The design constraint subsumes two conditions, $r_{cbdc} \geq -\left(1-\theta\right)\rho^{-1}$ and $\theta > \rho\left(r_d - r_{cbdc}\right)$, which respectively rule out the strict dominance of CBDC by cash and deposits (i.e., ensure that neither cash nor deposits offer all households a strictly better utility than CBDC) as per (5) and (7). For example, a completely cash-like CBDC ($\theta = 1$) that pays negative rates ($r_{cbdc} < 0$) would violate the first condition, such that all households have a strict preference for cash over CBDC. Because of network externalities, these conditions are necessary, but not sufficient, for positive CBDC take-up.

on bank intermediation (see Section 2.2.2), would remain.[35]

From an applied perspective, the focus on a single CBDC is well founded, as currently all central banks' CBDC studies center on the introduction of one CBDC.[36]

2.2 Equilibrium and welfare

A competitive equilibrium where deposit, loan and capital markets clear, is given by

$$s_d = d = l = k \tag{16}$$

The interaction of network externalities with the three money types in our framework leads to a rich set of equilibrium types, as shown in Table 1. For the sake of brevity, we introduce the parameter restrictions

$$1 < (A - \phi) \leq \frac{5}{2}; \quad \frac{3}{4} \leq \rho \leq \frac{3}{2}; \quad \underline{s} \leq \frac{1}{17} \tag{17}$$

which allow us to focus our discussions on "well-behaved" equilibria that give rise to plausible outcomes. For instance, when there is no CBDC, we should observe that deposits and cash are able to coexist, as they do in most countries.[37,38] We also bring forward a number of results based on optimal CBDC design that are formally derived later in the paper. Lemma 1 shows that an optimally designed CBDC always increases welfare. Therefore, the central

[35] For example, in a world with two CBDCs, the design of the most deposit-like CBDC determines the impact on bank intermediation, while the design of the most cash-like CBDC determines the intensity of competition with cash and hence whether cash falls prey to network effects.

[36] For an overview of ongoing CBDC developments, including links to central bank studies, see Auer et al. (2020) and Annex 1 within Kiff et al. (2020).

[37] While our model is not quantitative in nature, empirical evidence suggests that network effects only begin to play a significant role when the use of a payment instrument becomes very small, as represented by $\underline{s} \leq \frac{1}{17}$. For instance, in Canada, cash is widely accepted although only about 10 percent of transactions in value terms are conducted with cash (Engert et al., 2018). In contrast, in Sweden, where network effects on cash are becoming a source of concern, cash use stands near 1 percent of transactions value (Sveriges Riksbank, 2017). We discuss the outcome when cash demand is too low to sustain cash, even absent the introduction of CBDC, at the end of Section 3.1.

[38] The restriction $(A - \phi) > 1$ ensures that aggregate output (and hence consumption) increases in financial intermediation in equilibrium. This follows directly from the derivative $\frac{dY}{dk}$, which, given $k \leq 1$, is always positive for $(A - \phi) > 1$.

Table 1: **Possible equilibria and role in discussion**

Equilibrium	**Role in discussion**
Deposits & Cash & CBDC	Referred to as *ce*
Deposits & CBDC	Referred to as *nce*
Deposits & Cash	Never occurs under optimal policy
CBDC & Cash	Never occurs under optimal policy
CBDC only	Never occurs under optimal policy
Cash only	Impossible under any policy
Deposits only	Impossible under any policy

bank always prefers to introduce a CBDC. Lemma 2 shows that the CBDC design constraint in (15) does not bind, while deposits remain in use under an optimal CBDC design.

Lemma 1 *Under optimal policies derived in Section 3, the introduction of an optimally-designed CBDC always strictly raises social welfare.*

Proof. Provided in Appendix A. ■

Lemma 2 *Under optimal policies derived in Section 3, the parameter restrictions in (17) imply that $s_d \geq \underline{s}$ and $s_{cbdc} \geq \underline{s}$.*

Proof. Provided in Appendix A. ■

As indicated by Table 1, these Lemmas allow us to narrow the set of possible equilibria to just two well-behaved equilibria: a 'cash equilibrium' (denoted by *ce*) where all three money types are in use, and a 'cashless equilibrium' (denoted by *nce*) where cash disappears and only deposits and CBDC remain in use.[39]

Cash equilibrium In the cash equilibrium, cash use remains high enough to prevent network externalities from causing its disappearance. Using the properties of the uniform

[39] The three equilibria referred to as never occurring under optimal policy are further discussed in Appendix C.4, which considers outcomes of suboptimal CBDC design. The equilibria referred to as "impossible under any policy" are ruled out by the parameter restrictions which imply that, when there is no CBDC, the lowest possible shares of deposits and cash, respectively, are $\inf s_d = \frac{7}{22}$ and $\inf s_c = \frac{3}{22}$, both of which are above $\underline{s}$. The derivations of these results are available upon request.

distribution, (5)-(7) can be solved to obtain the shares of households holding each money type, which are

$$s_c^{ce} = \frac{1-\theta-\rho r_{cbdc}}{2} \tag{18}$$

$$s_d^{ce} = \frac{\rho\left(r_d - r_{cbdc}\right)+\theta}{2} \tag{19}$$

$$s_{cbdc}^{ce} = \frac{1-\rho r_d}{2} + \rho r_{cbdc} \tag{20}$$

Cashless equilibrium In the cashless equilibrium, network externalities lead to the disappearance of cash such that $s_c^{nce} = 0$, in which case the shares of households holding CBDC and deposits become

$$s_d^{nce} = \frac{\rho\left(r_d - r_{cbdc}\right)+\theta}{2} \tag{21}$$

$$s_{cbdc}^{nce} = 1 - \frac{\rho\left(r_d - r_{cbdc}\right)+\theta}{2} \tag{22}$$

Observe from (19) and (21) that the expressions for the shares of deposit holders are the same in the two equilibrium types ($s_d^{ce} = s_d^{nce}$). This is because, when there is a CBDC, deposits do not directly compete with cash. By (16), the expressions for deposit (and loan) interest rates, firm production and aggregate consumption are also the same in both equilibrium types and given by

$$r_d = \frac{2\left(A-1-\phi\right)-\left(\theta-\rho r_{cbdc}\right)}{2+\rho} \tag{23}$$

$$s_d = \frac{\rho\left(A-1-\phi-r_{cbdc}\right)+\theta}{2+\rho} \tag{24}$$

$$Y = \left(A - \frac{s_d}{2}\right) s_d \tag{25}$$

$$\int_{h\in[0,1]} C_{j^*(h)} dh = 1+\phi k_0 + \left(A-\phi-1-\frac{s_d}{2}\right) s_d \tag{26}$$

where s_d represents the share of deposits after substituting for r_d. Notably, regardless of the equilibrium type, the introduction of a CBDC that competes closely with deposits through a deposit-like design (i.e., low θ) and/or by offering high interest rates r_{cbdc} crowds out bank deposits. Although banks partially offset this by offering higher deposit rates r_d, in equilibrium this brings about a decline in bank intermediation, which also reduces firm production and aggregate consumption as per (25) and (26). The extent of the decline in aggregate consumption depends on relationship lending frictions. When these frictions are stronger (i.e., $A - \phi$ is larger), a given decline in s_d leads to a larger fall in aggregate consumption. This is precisely why we refer to $(A - \phi)$ as the value added from bank intermediation.

Lastly, it is important to note that network externalities lead to strategic complementarities in households' payment decisions, thus bringing about the possibility of multiplicity between cash and cashless equilibria. However, both of these equilibrium types may also arise due to fundamentals, and multiplicity does not lead to insights that are interesting, or that we observe in reality. Therefore, we rule out multiplicity by assuming that it is always resolved in favor of a cash equilibrium, which we consider to be similar to the pre-digital currency economy. A cashless equilibrium then arises only when fundamentals are such that the cash equilibrium is not self-confirming, which is the case when the boundary condition

$$s_c^{ce} \geq \underline{s} \tag{27}$$

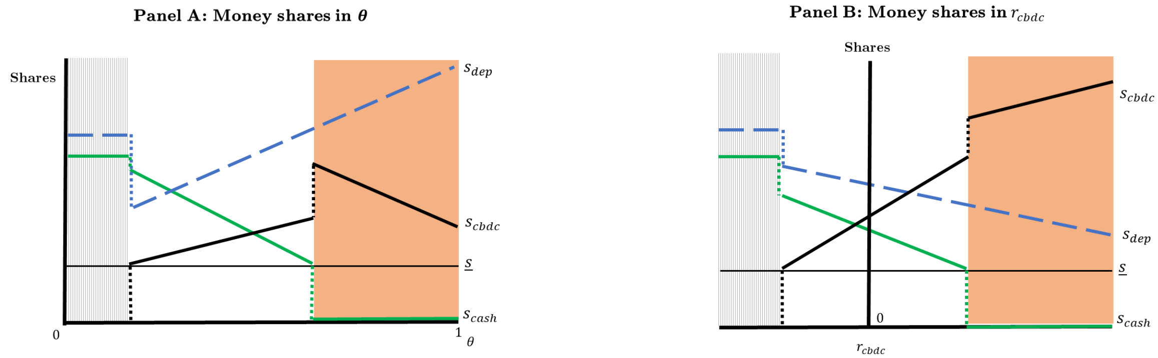
is violated.[40] Using (18), we can also write this condition in terms of CBDC design parameters as

$$\theta + \rho r_{cbdc} \leq 1 - 2\underline{s} \tag{28}$$

which indicates that a CBDC that competes strongly with cash through a cash-like design (i.e., high θ) and/or by offering a sufficiently high interest rate r_{cbdc} may eliminate cash and give rise to a cashless equilibrium. We proceed with a discussion of the resulting comparative statics.

[40]Resolving multiplicity in favor of the cashless equilibrium shifts the boundary condition to $\theta + \rho r_{cbdc} > 1 - 2\underline{s} - g(0)$ without any qualitative impact on our analysis.

Figure 1: **Comparative statics of money shares**



Note: Shares and boundaries are drawn according to (18)-(22) and (28). In Panel A, a more cash-like CBDC increases the shares of CBDC and deposits, while cash use declines. Cash falls out of use below a critical mass. In Panel B, when the CBDC interest rate increases, cash and deposits' shares fall. Conversely, when CBDC interest rates are too low, there is no CBDC take up.

2.2.1 Comparative statics

Figure 1 depicts the comparative statics of money shares in terms of CBDC design parameters (θ, r_{cbdc}). The unshaded part of Panel A shows that cash holdings decline and bank deposits rise as the CBDC becomes more cash-like with higher θ. Notably, the share of CBDC holders rises as the CBDC becomes more cash-like. This is because banks respond to reduced competition from CBDC by lowering deposit rates r_d, whereas cash offers no interest. However, when CBDC becomes so cash-like that cash use declines below a threshold $\underline{s}$, network externalities lead to a cashless equilibrium, depicted by the shaded area at the right end of Panel A. This leads to a jump in the use of the CBDC, because cash holders switch to it. As the CBDC becomes even more cash-like, households with preferences on the margin between CBDC and bank deposits switch to the latter, thereby raising deposits and reducing CBDC use. Panel B shows that a higher CBDC rate r_{cbdc} reduces the shares of both cash and deposits, while raising that of CBDC. However, as banks raise deposit rates in response to higher CBDC rates, deposits decline less than cash. A sufficiently high r_{cbdc} leads to a cashless equilibrium, which is depicted by the shaded area at the right end of the panel. Furthermore, the striped areas at the left ends of Panel A and B indicate domains where the CBDC design constraint is violated, and CBDC falls out of use.

Finally, note that we can analyze the impact of introducing a CBDC with a given design (θ, r_{cbdc}) by comparing it with the equilibrium under a CBDC that is completely cash-like $(\theta = 1)$ and offers no interest $(r_{cbdc} = 0)$. With this design, the CBDC is identical to cash and becomes completely innocuous in our model. In other words, introducing a CBDC is like moving θ from 1 to a lower value and/or changing r_{cbdc} away from 0. Once the CBDC moves away from cash mimicry, the combination (θ, r_{cbdc}) needs to be competitive enough if the CBDC is to have a positive uptake (as represented by the CBDC design constraint). Moreover, as compared to a world without CBDC, any such positive uptake of CBDC necessarily derives from *both* cash and deposits in our model. That is, introducing a CBDC always brings about some decline in banks' deposit base and consequently in bank intermediation to firms, although the extent of this effect depends on how closely the CBDC competes with deposits. To the extent that the CBDC reduces bank intermediation, it also causes a decline in aggregate output and consumption.

2.2.2 Welfare analysis

In equilibrium, social welfare can be split into two terms

$$W = \rho \int_{h \in [0,1]} C_{j^*(h)} dh - \int_{h \in [0,1]} \left| x_{j^*(h)} - h \right| dh \tag{29}$$

The first term represents aggregate consumption as given by (26). Because of the role of banks in providing firm financing, aggregate consumption relates closely to the extent of bank intermediation, and therefore to bank deposits s_d. The second term $\left| x_{j^*(h)} - h \right|$ represents welfare losses due to the distance between households' payment preferences and their preferred instrument. This term embodies the social value of variety in payment instruments, as increased variety provides heterogeneous households with greater opportunity to minimize the distance to their payment preferences.

How these two terms affect welfare, particularly in relation to CBDC design instruments

θ and r_{cbdc}, becomes clearer in the closed form expressions for welfare provided in Lemma 3.

Lemma 3 *Social welfare in the cash and cashless equilibria are respectively given by*

$$W^{ce}(\theta, r_{cbdc}) = \frac{4\rho\left(A-\phi-\frac{1}{2}\right)\theta + 4(1-\theta)\theta - 3\rho\theta^2 - (4+\rho)\rho^2 r_{cbdc}^2 + \omega_1}{4(2+\rho)} \quad (30)$$

$$W^{nce}(\theta, r_{cbdc}) = \frac{2\rho(A-\phi)\theta + (4-3\theta)\theta - 2\rho\theta^2 - \rho^2 r_{cbdc}^2 + \omega_2}{2(2+\rho)} \quad (31)$$

where ω_1 and ω_2 are collections of constants

$$\omega_1 \equiv \rho\left(7 + 4k_0\phi(2+\rho) + \rho\left(6 + 2A^2 - 4A(1+\phi) + 2\phi(2+\phi)\right)\right) - 2 \quad (32)$$

$$\omega_2 \equiv \rho^2\left(3 + A^2 - 2A(1+\phi) + \phi(2+2k_0+\phi)\right) + \rho(3+4k_0\phi) - 2 \quad (33)$$

Proof. Provided in Appendix A. ■

To provide intuition, we focus on the breakdown of the terms in the first expression (30) which pertains to welfare under the cash equilibrium, although a similar breakdown applies to (31) and the cashless equilibrium as well.

The first term $4\rho\left(A-\phi-\frac{1}{2}\right)\theta$ captures the relation between the value of bank intermediation $(A-\phi)$ and CBDC design characteristic θ. When $(A-\phi)$ is greater, aggregate consumption and social welfare depend more strongly on the intermediation derived from bank deposits. This, in turn, calls for a CBDC that competes less intensely with banks, through a more cash-like CBDC design (i.e., high θ).

The second and third terms, $4(1-\theta)\theta - 3\rho\theta^2$, pertain to the relation between variety in payment instruments and CBDC design. Notably, these terms are linear-quadratic in θ, meaning they have an interior maximum, and this captures the fact that variety in payment instruments is best served by an intermediate CBDC design that is differentiated from both cash and deposits.

The final term $-(4+\rho)\rho^2 r_{cbdc}^2$, is negative and quadratic in r_{cbdc}, with the implication that a non interest-bearing CBDC maximizes welfare within a given equilibrium type. This is because an interest-bearing CBDC distorts households' payment choices away from the instrument closest to their payment preferences. As this affects households on the margin between CBDC and another instrument, these distortions rise at an increasing rate as r_{cbdc} moves further from zero. Moreover, unlike deposit rates, payment choice distortions caused by r_{cbdc} are not offset by a contribution to financial intermediation. While deposit rates reflect the surplus from increased bank lending and the resulting rise in firm production, CBDC rates are funded by lump-sum transfers that have no direct productive impact.

Lastly, it is important to note that by focusing on welfare within a given type of equilibrium, our discussion has so far abstracted from network effects and the associated equilibrium determination condition (28). Accounting for these effects, social welfare is given by

$$W\left(\theta, r_{cbdc}\right) = \begin{cases} W^{ce}\left(\theta, r_{cbdc}\right) \text{ if } \theta + \rho r_{cbdc} \leq 1 - 2\underline{s} \\ W^{nce}\left(\theta, r_{cbdc}\right) \text{ otherwise} \end{cases} \tag{34}$$

and CBDC design parameters (θ, r_{cbdc}) may lead to a switch from one equilibrium type to another. In the next section, we shed more light on optimal CBDC design, including the role of the CBDC interest rate, in the presence of network effects.

3 Optimal CBDC design

In this section, we analyze the optimal CBDC design which maximizes social welfare, proceeding in two steps. First, we investigate the optimal design of a non interest-bearing CBDC, and then how optimization of an interest-bearing CBDC differs from this.

This two-step approach is more than a matter of analytical convenience. In practice, most central banks appear to be constraining themselves to non interest-bearing CBDCs. This may be due to political economy considerations, such as concerns about a central bank liability that is held by the general public and can be made to pay negative interest.

Alternatively, there could be legal hurdles to an interest-bearing CBDC. For instance, the need to tax interest earnings may interfere with a desire to offer a degree of anonymity on the CBDC in certain jurisdictions (Engert and Fung, 2017). However, in many countries tax exemptions have been granted for a variety of government issued securities. For example, in the US, interest earned on municipal bonds is exempt from Federal and state taxes for residents of the issuing state. Households' earnings from CBDC interest rates could similarly be granted tax exempt status. Moreover, interest on a CBDC token could take the form of dual currency system with a crawling exchange rate peg between digital currency and cash.[41] A predetermined appreciation or depreciation path that is built into the digital currency, much like for instance the volume of Bitcoin creation over time was predetermined at the coin's inception, would be equivalent to accountless interest payments. As such, anonymity of CBDC interest payments could in principle be maintained.[42]

Using our framework, we can analyze the economic ramifications of constraining the CBDC to a non interest-bearing form.

3.1 Non interest-bearing CBDC

When a CBDC is required to be non interest-bearing ($r_{cbdc} = 0$), the design optimization problem is given by

$$\max_{\theta \in [0,1]} \{W(\theta, 0)\} \tag{35}$$

where $W(\theta, 0)$ is defined according to (34) and Lemma 3.[43] Accordingly, the optimal CBDC design in the cash and cashless equilibria can be solved to

$$\theta^{ce} = \frac{2 + \rho(2(A - \phi) - 1)}{4 + 3\rho} \tag{36}$$

$$\theta^{nce} = \frac{2 + \rho(A - \phi)}{3 + 2\rho} \tag{37}$$

[41] See Lilley and Rogoff (2020) and Agarwal and Kimball (2015, 2019) for further discussions on a dual currency system.

[42] Another option for anonymous CBDC interest payments is to charge a fee or offer a discount on a nameless CBDC payment card (e.g., a $100 CBDC card would cost $99 or $101).

[43] The design constraint (15) is slack under optimal policies as per Lemma 2.

where the parameter restrictions (17) ensure that θ^{ce} and θ^{nce} are well-defined on $[0,1]$.[44] The implication is that optimal policy leads to an interior CBDC design where CBDC's cash-likeness equates the marginal benefit on bank intermediation with marginal losses to payments system variety from moving too close to cash. Furthermore, combining these expressions with (30) and (31) shows that as long as network effects play no role, the cash equilibrium welfare dominates the cashless equilibrium, so that

$$W^{ce}\left(\theta^{ce},0\right)-W^{nce}\left(\theta^{nce},0\right)=\frac{\left(\rho\left(A-\phi-2\right)-1\right)^{2}}{2\left(3+2\rho\right)\left(4+3\rho\right)}>0 \tag{38}$$

where the inequality is strict because $\rho\left(A-\phi-2\right)<1$ given our parameter assumptions. This dominance of the cash equilibrium derives from the fact that payment instrument variety creates social value for heterogeneous households. If sustaining that variety is costless, welfare is best served by having all three payment instruments in use.

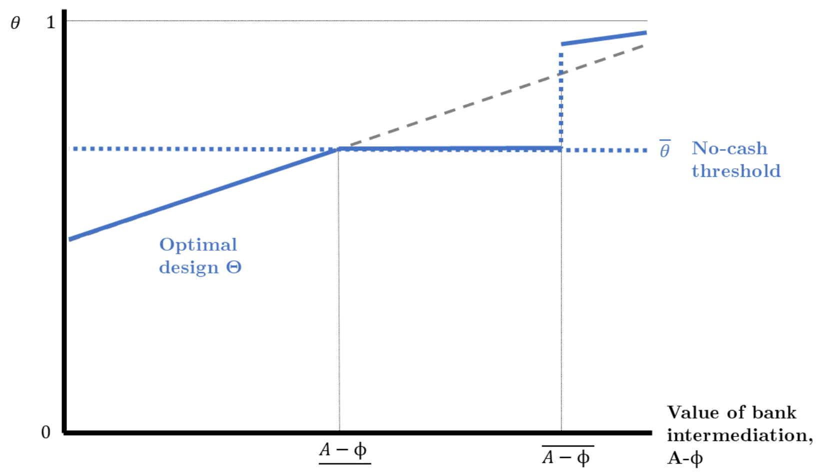
However, network effects impose costs on maintaining cash use by "constraining" the CBDC design optimization. In particular, when applied to a non interest-bearing CBDC, the condition (28) can be written as an upper bound on the degree to which the CBDC approximates cash,

$$\bar{\theta}\leq 1-2\underline{s} \tag{39}$$

above which the economy moves to a cashless equilibrium. That is, whenever $\theta^{ce}>\bar{\theta}$, the choice is no longer between $W^{ce}\left(\theta^{ce},0\right)$ and $W^{nce}\left(\theta^{nce},0\right)$, since the latter is no longer implementable: A CBDC with design θ^{ce} is too cash-like and would reduce cash use below the threshold $\underline{s}$ where network effects cause cash demand to spiral down to zero. Instead, optimization centers on $W^{ce}\left(\bar{\theta},0\right)$ versus $W^{nce}\left(\theta^{nce},0\right)$, that is, preserving the cash equilibrium under a CBDC design constrained at $\bar{\theta}$ versus allowing cash to vanish and having unconstrained optimal policy with only CBDC and deposits in existence. Defining $\boldsymbol{\Theta}$ as optimal policy when taking into account the constraint imposed by network effects, we obtain Figure

[44] Given (17), these optimal policies can range between $\theta^{ce}\in\left[\frac{7}{17},\frac{16}{17}\right]$ and $\theta^{nce}\in\left[\frac{7}{12},\frac{23}{24}\right]$.

Figure 2: **Optimal non interest-bearing CBDC design**



Note: The CBDC is optimally made more cash-like when bank intermediation matters more. But at cash' critical threshold, optimal design is distorted towards deposit-like to retain cash. For high enough intermediation value, cash is allowed to disappear and the CBDC becomes more cash-like to compensate.

2.[45]

Figure 2 brings together several key aspects of our model.[46] First, when network effects do not constrain policy, the optimal similarity of the CBDC to cash depends on the extent to which banks have an advantage at alleviating financial frictions. On the one hand, locating the CBDC "centrally" relative to the attraction points of deposits and cash serves the payment needs of households with diverse preferences. On the other hand, when bank intermediation has more value, the CBDC is optimally made more cash-like, so as to limit its adverse impact on banks' deposit base, and thereby on aggregate output and consumption.

Second, as the value of bank intermediation $(A - \phi)$ rises, a threshold $\underline{A - \phi}$ is eventually reached, beyond which optimal design freezes in relation to $(A - \phi)$. This is because optimal policy prevents the disappearance of cash in order to protect payment instrument variety. As long as the welfare gains from this variety outweigh the welfare costs from lost bank inter-

[45] This is formally derived in Proposition 1 below.

[46] In addition to optimal policy derived in the Proof of Proposition 1, the exact shape of Figure 2 relies on two more properties from (36) and (37): first, $\theta^{nce} > \theta^{ce}$; second, $\frac{\partial \theta^{ce}}{\partial (A-\phi)} < \frac{\partial \theta^{nce}}{\partial (A-\phi)} < 0$ and therefore the slope of θ^{nce} is flatter.

mediation, optimal policy maintains all three payment instruments, rather than tipping cash over the disappearance point induced by network effects. However, when preserving bank intermediation becomes the dominant concern (namely when $A - \phi$ exceeds the threshold $\overline{A - \phi}$), optimal policy forgoes variety, allowing cash to disappear, in exchange for a larger deposit base for the banks.

Third, once cash vanishes, the CBDC bears the brunt of servicing former cash users, and therefore optimally moves further towards cash than it would have if all three forms of money were still in existence. In Figure 2, this is seen from the portion of the blue line to the right of $\overline{A - \phi}$, which is above the dashed gray line.

This last portion of the blue line also demonstrates the outcome when fundamentals are such that cash disappears prior to the introduction of a CBDC. If the CBDC is introduced after cash has disappeared, optimal policy is simply depicted by extending the portion of the blue line to right of $\overline{A - \phi}$ all the way to the vertical axis. In this case, network effects no longer play a role since cash would already be in disuse. As such, in a country that starts off cashless, optimal CBDC policy is quite straightforward, and the CBDC is always more cash-like than when cash exists.

3.2 Interest-bearing CBDC

This section considers an interest-bearing CBDC, where the CBDC rate can be varied as desired to maximize welfare. Unconstrained optimal policy is now given by the solution to the system of two first-order conditions, $\left\{\frac{\partial W(\theta, r_{cbdc})}{\partial \theta} = 0, \frac{\partial W(\theta, r_{cbdc})}{\partial r_{cbdc}} = 0\right\}$, which yields the same expressions for θ^{ce} and θ^{nce} as in (36) and (37) and for CBDC rates

$$r_{cbdc}^{ce} = r_{cbdc}^{nce} = 0 \tag{40}$$

with the implication that the optimal CBDC rate is always zero in the absence of network effects. This outcome is consistent with the discussion in Section 2.2.2, which suggest that the CBDC interest rate is a suboptimal tool compared to θ. As such, our model indicates that in a world without network effects, central banks would be right to focus their attention on the issuance of non interest-bearing CBDCs.[47]

However, as discussed in Section 3.1, such an optimal policy profile is not always implementable, due to network effects. For an interest-bearing CBDC, the equilibrium determination condition (28) affects both θ and r_{cbdc}. When this condition binds, and the central bank chooses to satisfy it in order to preserve cash, optimal CBDC design becomes

$$\widetilde{\theta} = \frac{2+\rho\left(2\left(A-\phi\right)-1-\rho\left(4+\rho\right)\left(2\underline{s}-1\right)\right)}{4+\rho\left(1+\rho\right)\left(3+\rho\right)} \tag{41}$$

$$\widetilde{r}_{cbdc} = -2\frac{\rho\left(\left(A-\phi\right)+3\underline{s}-2\right)+4\underline{s}-1}{4+\rho\left(1+\rho\right)\left(3+\rho\right)} \tag{42}$$

Hence, when network effects come into play, the optimal CBDC rate diverges from zero. Indeed, under the parameter restrictions in (17), the optimal CBDC rate always turns negative. This in turn allows CBDC design to become more cash-like than in the non-interest bearing case ($\widetilde{\theta} > \overline{\theta}$), since the value of bank intermediation $(A-\phi)$ rises.

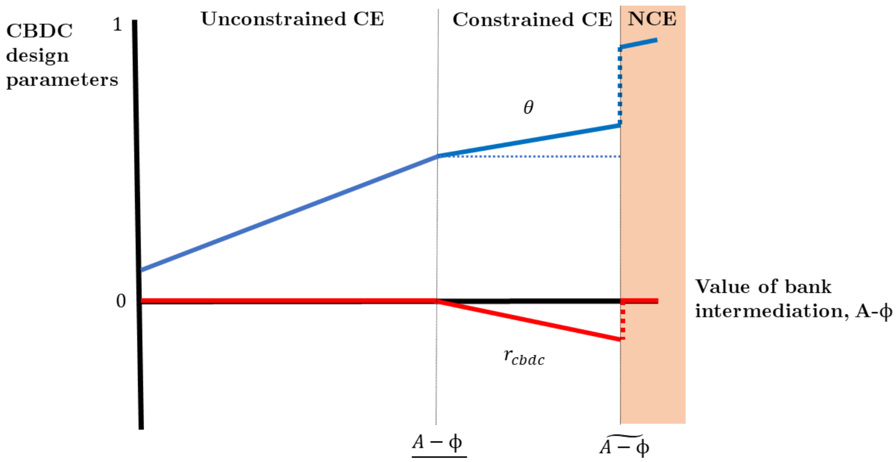
Note that the constrained non interest-bearing optimal policy $(\theta, r_{cbdc}) = \left(\overline{\theta}, 0\right)$ is within the feasible set of policies delineated by (28), but is found to be sub-optimal. Therefore, access to a second policy tool in the CBDC interest rate strictly raises welfare when network effects bind. Proposition 1 records our key results on optimal CBDC design, which are depicted in Figure 3.

Proposition 1 *There is a cutoff $\overline{\rho} \in \left(\frac{3}{4}, \frac{3}{2}\right)$, such that when $\rho < \overline{\rho}$, cash never vanishes under an optimally designed interest-bearing CBDC, where optimal design is given by*

$$\begin{cases} \left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right) & \text{if } \theta^{ce} + \rho r^{ce}_{cbdc} < 1 - 2\underline{s} \\ \left(\theta^{ce}, r^{ce}_{cbdc}\right) & \text{otherwise} \end{cases} \tag{43}$$

[47] Appendix C investigates the robustness of this key result. We find that the optimality of zero CBDC rates (absent network effects) is robust to the specification of the production function. However, when banks have market power (Appendix C.3), or when anonymous payment instruments create negative social externalities (Appendix C.2), the optimal CBDC rate can deviate from zero.

Figure 3: **Optimal interest-bearing CBDC design**



Note: The central bank jointly determines the CBDC's design and interest rate. The CBDC interest rate is used when network effects bind, which makes it easier to sustain payment variety: the threshold where cash vanishes is pushed out.

For $\rho > \overline{\rho}$, cash can vanish when the value of bank intermediation exceeds the threshold $\widetilde{A-\phi}$. However, this threshold is higher when the CBDC bears interest than when it does not, that is, $\widetilde{A-\phi} > \overline{A-\phi}$

Proof. Provided in Appendix A. ■

When the relative weight of payment preferences in household utility is large enough ($\rho < \overline{\rho}$), the presence of a variable CBDC interest rate as a second tool fundamentally alters the outcomes under optimal policy, as compared to a non interest-bearing CBDC, depicted in Figure 2. With a non interest-bearing CBDC, the only means to safeguard deposits is to make the CBDC eat into cash demand. But with a variable CBDC interest rate, optimal policy simultaneously reaps the welfare benefits of sustaining variety in payment instruments and limits bank disintermediation. In particular, when network effects bind, optimal policy combines a (more) cash-like CBDC with a negative CBDC interest rate, thereby avoiding adverse network effects on cash use by making the CBDC less attractive, while simultaneously limiting the CBDC's impact on financial intermediation and production. This optimal policy is portrayed in the unshaded part of Figure 3. For $\rho < \overline{\rho}$, the shaded part of this figure is never reached, which means that cash never vanishes under optimal policy.

However, the deeper the CBDC interest rate moves into negative territory, the larger its costs in terms of payment choice distortions become. If the weight on payment preferences is relatively small ($\rho > \overline{\rho}$), then a point is reached where the value of bank intermediation ($A - \phi$) is large enough that welfare is best served by letting go of cash. This case, portrayed by the shaded area in Figure 3, is similar to the jump seen in Figure 2: optimal policy switches to ($\theta^{nce}, r_{cbdc}^{nce}$), which implies a more cash-like CBDC to better accommodate the preferences of previous cash users, and a return to zero CBDC rates. Nevertheless, even when $\rho > \overline{\rho}$, the availability of CBDC interest rates serves a purpose. In particular, raising $(A - \phi)$ from lower to higher values, the possibility of varying the CBDC interest rates "delays" the jump to a cashless equilibrium where households lose access to three differentiated means of payment (i.e., $\widetilde{A - \phi} > \overline{A - \phi}$)

3.3 Distributional effects

So far, our welfare analysis has centered on aggregate welfare, which represents the total utility of all households. Introducing an optimally designed CBDC always raises aggregate welfare in our framework, but this is far from a Pareto improvement: some households gain while others lose.

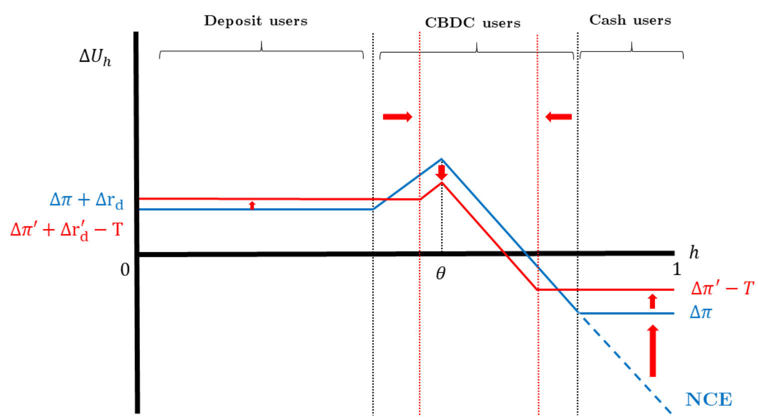
3.3.1 The CBDC's impact on different households

Figure 4 shows the welfare impact of introducing a CBDC across the distribution of household preferences $h \in [0, 1]$.[48] The blue line depicts the impact of a non interest-bearing CBDC.

To begin with, households with payment preferences closest to deposits (i.e., low h) remain as deposit users after the introduction of a CBDC. These households are impacted by the introduction of a CBDC through its negative effects on financial intermediation, as well as its positive effects on bank deposit rates. On the one hand, the decline in financial intermediation reduces total production and therefore profit transfers, π, from firms. On the

[48] See Appendix B for the underlying derivations.

Figure 4: **Distributional effects of CBDC**



Note: An optimally designed CBDC raises aggregate welfare, but not all households gain. Cash holders lose, especially if cash vanishes. A negative CBDC interest rate redistributes gains from CBDC users to others. See Appendix B for the underlying derivations.

other hand, CBDC competition with bank deposits drives up deposit rates, r_d. Overall, the latter effect dominates and the introduction of a CBDC raises the consumption, and hence the welfare, of all deposit users.

At the other end of the spectrum, households with a strong preference for anonymity (i.e., high h) continue to use cash. CBDC impacts the welfare of these households through consumption. Since cash does not pay interest, the decline in firm profits π brings about a decline in consumption and welfare for these households. If instead the CBDC instigates network effects on cash and drives it out of use, these households suffer from a further decline in welfare due to the loss of their preferred payment instrument. The extent of their welfare loss then becomes proportionate to their preference for anonymity, as depicted by the dashed blue line.

Households that switch from deposits to CBDC experience a net welfare gain from the introduction of the CBDC, since otherwise they would have continued to use deposits. In other words, by virtue of optimality, these households only switch to CBDC if the gains in terms of payment preferences outweighs the loss of interest payments r_d. The household with preference $h = \theta$ experiences the greatest increase in welfare. For households that marginally prefer CBDC to cash, the net welfare effect is negative, since CBDC holders also suffer from a fall in consumption due to reduced firm profits.

Overall, we can define a boundary household, $\overline{h}$, such that households $h \in [0, \overline{h})$ gain from the introduction of a CBDC and households $h \in (\overline{h}, 1]$ lose, where the gains of the former group more than offset the losses of the latter in the aggregate. The fact that depositors emerge as winners and cash holders as losers hints at a potentially regressive impact of a CBDC. In our analysis, all households have identical endowments. In practice, however, households that primarily conduct their payments with cash tend to have lower income, while higher income households more often rely on deposit-based payments.

The red line in Figure 4 shows the welfare impact of an interest-bearing CBDC with slightly negative CBDC rates as per the optimal design prescribed in Section 3.2. Three factors determine the impact of negative CBDC rates here. First, the revenues from negative CBDC rates are transferred lump-sum to all households, which effectively redistributes welfare gains from CBDC users to cash and deposit users. Second, since negative CBDC rates increase deposits and financial intermediation, firm profits π rise, which benefits all households, while deposit rates r_d decrease, hurting depositors. However, the second effect is dominated by the first, in that overall CBDC users lose out and deposit and cash users gain from the CBDC rate cut. Third, when CBDC rates prevent cash from going out of use, they stave off large welfare losses for cash holders from the loss of a preferred payment instrument.

3.3.2 Fiscal transfers and Pareto improvement

Since the introduction of a CBDC raises aggregate welfare but lowers the welfare of some households, a social planner could use lump sum taxes and subsidies to try and ensure that all households are better off from the CBDC. This is straightforward to accomplish if the social planner is capable of conditioning transfers on household preferences. However, in practice, the planner is unlikely to know individual household preferences. Instead, the planner might be able to observe households' selected payment instruments, for example from the way that households choose to pay their taxes.

We show that the social planner cannot make the introduction of a CBDC into a Pareto improvement by conditioning transfers on observed modes of household payments. The planner is well placed to redistribute between payment categories in which all households have become better off or worse off due to the CBDC. Indeed, households that continue using deposits are all better off, whereas households that continue using cash are all worse off, and therefore the planner could potentially redistribute gains from one group to the other. But households that switch to using CBDC do not all experience the same welfare effect. As discussed above, $\overline{h}$ is within the part of the spectrum where households switch to CBDC, meaning that some of those households are better off, while some are worse off. Therefore, the only way that the planner could achieve the desired Pareto improvement is by compensating all CBDC users sufficiently that the least well off CBDC user is as well off as before the introduction of the CBDC. That is, deposit users would have to be taxed enough to achieve this, and to compensate cash users, without being taxed so heavily that they are themselves worse off than before the introduction of a CBDC. We show in Appendix B.4 that this is not possible.

In sum, because part of the welfare gain from the introduction of a CBDC is captured by a subset of households that switch to using CBDC, and the social planner cannot differentiate these from other households that switch to using CBDC, a transfer mechanism to make everyone better off from the introduction of a CBDC cannot be engineered.[49]

[49] Note that tax distortions (i.e., if households know in advance that redistributions based on their means of payment choice will lead to transfers) could further complicate the social planner's task. But even abstracting from this (i.e., assuming no such foreknowledge on households' part), as we do here, does not suffice to make the Pareto improvement feasible.

4 Conclusion

As central banks across the world weigh the introduction of a digital currency, the implications of a CBDC for money demand and financial intermediation are coming to the fore. This paper relates the effects on cash, deposits and bank intermediation to two key design choices involved in developing a CBDC: the degree to which the CBDC resembles cash, and whether it bears interest. In our framework, the social value of the CBDC comes from the fact that it can bring some of the anonymity of cash into the digital realm. The demand for digital payment privacy is already a major issue in some jurisdictions, and is likely to gain increased prominence globally with the spread of fintech and companies' ability to parse transactions data for their own gain.

The CBDCs currently under consideration are mostly of a non interest-bearing type. Analyzing the optimal design of a non interest-bearing CBDC exposes a challenging welfare tradeoff for the central bank. On the one hand, a cash-like CBDC risks reducing the demand for cash below the critical mass where ATMs become sparser and fewer shops accept cash, placing at risk the variety of payment instruments that is valuable to households with diverse needs. On the other hand, if the central bank makes a CBDC more similar to deposits, the banks' deposit base can come under threat, with negative implications for credit provision and output, especially if banks have a significant role in alleviating lending frictions.

Overall, in an economy where the banks' role is limited, a CBDC is best designed in a manner that is as distinct from existing payment instruments as possible. Greater focus on preserving bank intermediation instead drives the optimal design of a CBDC to be more cash-like, but only up to a point: concerns that cash may fall prey to network effects gives the central bank cause to limit the extent to which CBDC competes against cash. Only when conserving banks' deposit base becomes the overarching concern does the central bank give up on cash, and optimal policy then jumps towards a more cash-like CBDC.

When network effects matter, an interest-bearing CBDC helps the central bank alleviate these tradeoffs. Moving the CBDC interest rate away from zero causes welfare losses as it creates price distortions in households' choice between payment instruments. As long as network effects do not hold sway, the central bank thus shies away from varying the CBDC interest rate. Therefore, in a world where network effects have no material impact, nothing is lost by limiting CBDC design to non interest-bearing CBDCs. However, when network effects pose a threat to the variety of payment instruments, an interest-bearing CBDC becomes optimal. Notably, provided households care enough about payment variety, the CBDC interest rate can be used to ensure that cash remains in use. That is, an optimally designed interest-bearing CBDC meets the aims of safeguarding bank intermediation and protecting the trio of payment instruments against network effects, irrespective of the role of financial frictions in the economy.

This finding provides an economic counterweight to the political economy considerations that may otherwise drive central banks to opt for a non interest-bearing CBDC, such as concerns about the possibility of negative rates on publicly accessible central bank liabilities. At this early stage, when CBDCs are still in the laboratory, central banks may want to at least keep an eye on the inclusion of an adjustable CBDC interest rate, weighing its benefits against possible political economy costs.

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Appendices

This section contains the following appendices:

A Proofs

Proof of Lemma 1. A CBDC can be designed in a manner that mimics cash: $(\theta, r_{cbdc}) = (1, 0)$. From this, it directly follows that welfare in both ce and nce is higher than in an equilibrium without CBDC: in both ce and nce the central bank could attain the same welfare as in the equilibrium without CBDC, by setting $\theta = 1$ and $r_{cbdc} = 0$, but this policy combination is never optimal, as seen from (36) and (37) where $\theta^{ce} < \theta^{nce} < 1$. Hence, $W(1,0) < W(\theta^{nce}, r^{nce}_{cbdc}) < W(\theta^{ce}, r^{ce}_{cbdc})$, where the last inequality follows from (38). ■

Proof of Lemma 2. Substituting (36), (40) and (23) into (18)-(20) gives the expressions for the shares of money, s^{ce}_c, s^{ce}_d, and s^{ce}_{cbdc}, when all forms of money exist (ce), in terms of parameters only. We can then calculate the infima of s^{ce}_c, s^{ce}_d, and s^{ce}_{cbdc} over the parameter space defined by (17). This yields

$$
\begin{aligned}
\inf s^{ce}_c &= \frac{1}{34} \\
\inf s^{ce}_d &= \frac{2}{17} \\
\inf s^{ce}_{cbdc} &= \frac{1}{17}
\end{aligned}
$$

and therefore, given $\underline{s} \leq \frac{1}{17}$ in (17), it follows that $\eta_d = \eta_{cbdc} = 0$.[50]

Moreover, using (36) and (40), as well as (23), we can also verify that two necessary conditions for positive uptake of the CBDC, which are subsumed by the CBDC design constraint (15), are also satisfied. These conditions are

$$r_{cbdc} \geq -(1-\theta)\rho^{-1} \tag{44}$$

$$\theta > \rho(r_d - r_{cbdc}) \tag{45}$$

which respectively rule out the strict dominance of CBDC by cash and deposits (i.e., ensure that neither cash nor deposits offer all households a strictly better utility than CBDC) as

[50] This also remains valid in nce where $\inf s^{ce}_d = \frac{1}{6}$ and $\inf s^{ce}_{cbdc} = \frac{1}{12}$.

per (5) and (7). First, since $\sup\theta^{ce} = \frac{16}{17} < \sup\theta^{nce} = \frac{23}{24} < 1$, while $r_{cbdc} = 0$, condition (44) cannot be violated. Second, as $\inf\left(\theta^{ce} - \rho r_d\right) = \frac{1}{17}$ over the parameter space in (17), (45) is never violated either (and this necessarily also holds for θ^{nce}, since $\theta^{nce} > \theta^{ce}$). ■

Proof of Lemma 3. $W^{ce}\left(\theta, r_{cbdc}\right)$ can be determined by solving the following system of 11 equations in 11 unknowns, which gives the expression in (30):

$$
\begin{aligned}
W &= \rho\left(1+\phi k_0+\left(A-1-\phi-\frac{s_d}{2}\right)s_d\right)-s_dE_d-s_{cbdc1}E_{cbdc1}-s_{cbdc2}E_{cbdc2}-s_cE_c\\
s_d &= \frac{\theta+\rho\left(r_d-r_{cbdc}\right)}{2}\\
s_{cbdc} &= s_{cbdc1}+s_{cbdc2}\\
s_{cbdc1} &= \frac{\theta-\rho\left(r_d-r_{cbdc}\right)}{2}\\
s_{cbdc2} &= \frac{1+\rho r_{cbdc}-\theta}{2}\\
s_c &= \frac{1-\theta-\rho r_{cbdc}}{2}\\
E_d &= \left|0-\frac{1}{2}s_d\right| = \frac{s_d}{2}\\
E_{cbdc1} &= \left|\theta-\left(s_d+\frac{1}{2}s_{cbdc1}\right)\right| = \left(\theta-s_d-\frac{1}{2}s_{cbdc1}\right)\\
E_{cbdc2} &= \left|\theta-\left(s_d+s_{cbdc1}+\frac{1}{2}s_{cbdc2}\right)\right| = s_d+s_{cbdc1}+\frac{1}{2}s_{cbdc2}-\theta\\
E_c &= \left|1-\left(1-\frac{1}{2}s_c\right)\right| = \frac{1}{2}s_c\\
r_d &= \frac{2\left(A-1-\phi\right)+\rho r_{cbdc}-\theta}{2+\rho}
\end{aligned}
$$

Similarly, the solution for $W^{nce}\left(\theta, r_{cbdc}\right)$ is found by setting $s_c = 0$ in the above, and solving. This yields the expression in (31). ■

Proof of Proposition 1. First, we note that $W^{ce}\left(\theta^{ce}, r^{ce}_{cbdc}\right) > W^{nce}\left(\theta^{nce}, r^{nce}_{cbdc}\right)$, per (38). Moreover, $W^{ce}\left(\theta^{ce}, r^{ce}_{cbdc}\right) > W^{ce}\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right)$ per definition, as welfare under unconstrained optimal policies exceeds welfare under constrained optimal policies within a given equilibrium (namely, ce). Hence, as long as the unconstrained ce is feasible, it is optimal. Therefore, the relevant comparison centers on $W^{nce}\left(\theta^{nce}, r^{nce}_{cbdc}\right)$ versus $W^{ce}\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right)$ when the network effects constraint matters, that is, when $\theta^{ce} + r^{ce}_{cbdc} = \theta^{ce} > 1 - 2\underline{s}$.

Second,

$$\inf_{A,\phi,\underline{s}} \left[W^{nce}\left(\theta^{nce}, r^{nce}_{cbdc}\right) - W^{ce}\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right)\right] > 0 \Leftrightarrow \rho > 1.431 = \overline{\rho} \tag{46}$$

which means that for $\rho < \overline{\rho}$, the policy combination $\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right)$ always (i.e., for any values of other parameters) welfare dominates $\left(\theta^{nce}, r^{nce}_{cbdc}\right)$, and hence cash never vanishes under optimal policies. Instead, for $\rho > \overline{\rho}$, there exist parameterizations, including the extremes of $(A - \phi) = \frac{5}{2}$ and $\underline{s} = \frac{1}{17}$, such that $\left(\theta^{nce}, r^{nce}_{cbdc}\right)$ welfare dominates $\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right)$. That is, when $\rho > \overline{\rho}$, cash can optimally be allowed to vanish, when network effects are strong enough ($\underline{s}$) and the value of bank intermediation $(A - \phi)$ is large enough.

Third, whenever $\theta^{ce} > \overline{\theta} = 1 - 2\underline{s}$, it is necessarily true that

$$W^{ce}\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right) - W^{ce}\left(\overline{\theta}, 0\right) > 0 \tag{47}$$

since $\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right) = \left(\overline{\theta}, 0\right)$ is within the possibility set of $\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right)$ but is not optimally chosen, as seen from (41) and (42). Hence, the range of parameter values where $W^{ce}\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right) > W^{nce}\left(\theta^{nce}, r^{nce}_{cbdc}\right)$ is broader than the range where $W^{ce}\left(\overline{\theta}, 0\right) > W^{nce}\left(\theta^{nce}, r^{nce}_{cbdc}\right)$. To put this in more concrete terms, consider $\rho > \overline{\rho}$ and $\underline{s} = \frac{1}{17}$. Then, the value of $(A - \phi)$ that is large enough to induce a switch from *ce* to *nce* is higher when policies are set at $\left(\widetilde{\theta}, \widetilde{r}_{cbdc}\right)$ than when they are set at $\left(\overline{\theta}, 0\right)$. ■

B Derivation of distributional effects

The foundations for Figure 4 are found by considering the impact of a CBDC on, respectively, deposit, cash and CBDC users. We use the term "after the introduction of a CBDC" to indicate the comparison between a world with cash and deposits only, and one where CBDC is available as an additional payment instrument.

B.1 Depositors

For a household that continues using deposits after the introduction of a CBDC, such as $h = 0$, nothing changes in terms of the payment preference aspect of utility through the introduction of a CBDC. Hence, tradeoff of that household centers on consumption, as represented by

$$C_d = 1 + r_d + \pi - T \tag{48}$$

$$= 1 + r_d (1 - s_d) + \phi k_0 + \left(A - 1 - \phi - \frac{s_d}{2}\right) s_d - r_{cbdc} s_{cbdc} \tag{49}$$

where $T = r_{cbdc} s_{cbdc}$ and π has been replaced using (10), (11), and (16). Further substituting for s_d, s_{cbdc}, and r_d with the expressions as shown in the proof of Lemma 3, this gives a closed-form expression for C_d. From this expression, we obtain

$$\left.\frac{\partial C_d}{\partial \theta}\right|_{r_{cbdc}=0} = \frac{\rho (A - \phi - 2) + \theta - 2}{(2 + \rho)^2} < 0 \tag{50}$$

which means that the introduction of a non interest-bearing CBDC always raises welfare for households that continue using deposits, because the introduction of a CBDC is equivalent to lowering θ from $\theta = 1$ (cash equivalence) to a lower value.[51] Put differently, the more intensely the CBDC competes with bank deposits (lower θ) the more it pushes up deposit

[51] Formally, we can verify that $\left.\frac{\partial C_d}{\partial \theta}\right|_{r_{cbdc}=0} < 0$ by noting that in (50) when $A - \phi \to 1$ the expression becomes $\frac{\theta - \rho - 2}{(2+\rho)^2}$ and when $A - \phi \to \frac{5}{2}$ it becomes $\frac{\frac{1}{2}\rho + \theta - 2}{(2+\rho)^2}$, both of which are smaller than 0 given $\rho \leq \frac{3}{2}$ and $\theta \leq 1$. Hence, $\left.\frac{\partial C_d}{\partial \theta}\right|_{r_{cbdc}=0} < 0$ always holds over the parameter space in (17).

rates, and the larger the welfare gains to depositors.

Moreover,

$$\frac{\partial C_d}{\partial r_{cbdc}} = \frac{-4 - \rho\left(4\left(A - \phi - 1\right) - \rho + 2r_{cdbc}\left(8 + \rho\left(5 + \rho\right)\right) + \theta\left(4 + \rho\right)\right)}{2\left(2 + \rho\right)^2} \tag{51}$$

where we find that at $r_{cdbc} = 0$, this term is negative overall, given the parameter space in (17) and $\theta \leq 1$. Hence, a marginal CBDC interest rate cut from $r_{cdbc} = 0$ to $r_{cdbc} < 0$ always raises the welfare of depositors.

B.2 Cash holders

For a household that continues using cash after the introduction of a CBDC (provided cash remains in use), such as $h = 1$, welfare effects similarly center on consumption only, as that household's preferences for payment instruments are unaffected. Contrary to depositors, however, the impact of a non interest-bearing CBDC on cash holders is straightforward: while depositors see gains from increased deposit rates that (more than) compensate for lost firm profit transfers, cash holders only experience those lost profit transfers, and are therefore necessarily worse off: $\frac{\partial C_c}{\partial \theta}\Big|_{r_{cbdc}=0} > 0$. Those cash holders would be even worse off if network effects push cash out of use and they are forced to take solace in a CBDC that is more distant from their payment preferences.

The impact of negative CBDC rates is also straightforward for cash holders. As cash pays no interest, the only channels through which cash holders are affected are π, which rises as the CBDC rate declines (increased financial intermediation), and T, which is positive when CBDC interest rates are negative (CBDC holders are taxed, and the proceeds accrue to all households). That is, $\frac{\partial C_c}{\partial r_{cbdc}} < 0$, as shown in Figure 4.

B.3 CBDC users

For households that switch to CBDC after it has been introduced, the key question is whether their gains in payment preferences outweigh lost consumption arising from bank disintermediation. Former depositors switching to CBDC, always see a welfare improvement overall. If they did not, they would have remained depositors, since depositors see welfare gains from the introduction of a CBDC, as per (50). The $h = \theta$ household experiences the largest welfare gain from the availability of a CBDC, because the CBDC precisely meets that household's payment preferences.

However, some of the $h > \theta$ CBDC holders would have been better off had CBDC not existed. After all, the household that is exactly indifferent between holding cash and holding CBDC experiences a welfare loss, since all cash holders lose welfare, and this household is indifferent between the welfare loss of continuing to hold cash, and the welfare loss from holding CBDC. CBDC holders with h marginally below this indifferent household would also certainly see an overall welfare loss. CBDC does not offer them enough of an attractive payment option to compensate for the loss in firm profit transfers. Finally, a negative CBDC rate acts as a tax on CBDC holders, and therefore reduces their welfare, as shown in Figure 4.

B.4 Social planner

Section 3.3.2 posed the question whether a social planner could design a set of lump-sum taxes and subsidies to households that would allow all households to be better off from the introduction of a (non interest-bearing) CBDC, when the planner can only differentiate between the households on the basis of their chosen means of payment.

As discussed in Section 3.3.2, CBDC users should be compensated sufficiently that the least well off CBDC user is no worse off than before the introduction of the CBDC. Per B.3, the least well off CBDC user is the one that is indifferent between CBDC and cash. This can also be seen from Figure 4. Because this household is indifferent between CBDC and cash, it must have experienced no utility gain from the new payment instrument, and it experiences the same welfare loss as cash holders (as can also be seen from Figure 4). As discussed in B.2, the loss of cash users in the case of a non-interest bearing CBDC is π. Therefore, the social planner needs to compensate all cash users and all CBDC users with a lump-sum subsidy sized π.

This subsidy has to come from lump-sum taxes on deposit users. Note that we can discuss compensation in purely monetary terms here as all relevant households for this comparison (cash users, the CBDC user that is indifferent between CBDC and cash, and deposit users) have experienced no change to their payment utility term from to the introduction of the CBDC. Moreover, from (48), for a non interest-bearing CBDC (where $T = 0$) the deposit users' welfare change concentrates only on the terms r_d and π.

Looking across all households, the gains from a higher r_d (which accrue to deposit users) have to compensate for the loss of π that all households experience. But r_d and π represent transfers from the perfectly competitive banking and firm sectors to households, and the total value of those transfers must decline, since bank intermediation has declined from the CBDC's impact on the bank's deposit base.[52] Therefore, no lump-sum transfer scheme can be designed which leaves all deposit users, all cash users, and the least well of CBDC user at least as well off as before the introduction of a CBDC.

[52] Also note that while r_d increases due to the introduction of a CBDC, this is only because there are fewer depositors left: $s_d r_d$ declines, meaning that total bank interest payments to all households are smaller, just as π is smaller, because total production has decreased.

C Extensions

C.1 Constant returns to scale production function

The baseline model considers a decreasing returns to scale (quadratic) firm production function. Here, we show that central components of the optimal policy profiles we derived, as represented by equations (36), (37) and (40), are robust to the use of a constant returns to scale production function. Instead of $Y = \left(A - \frac{k}{2}\right) k$, we now replace (10) with

$$Y = Ak \tag{52}$$

Following the same steps as in the main text, we obtain the following outcomes for optimal policies in *ce*

$$\theta^{ce} = \frac{1 + \rho\left(A - \phi - 1\right)}{2} \tag{53}$$

$$r_{cbdc}^{ce} = 0 \tag{54}$$

and in *nce*

$$\theta^{nce} = \frac{2 + \rho\left(A - \phi - 1\right)}{3} \tag{55}$$

$$r_{cbdc}^{nce} = 0 \tag{56}$$

Thus, the optimal unconstrained CBDC interest rate remains zero, in both *ce* and *nce*. Moreover, the CBDC is optimally made more similar to cash (i.e., to help preserve bank deposits) when the value of bank intermediation, $(A - \phi)$, rises.[53]

[53] Decreasing and constant returns to scale production functions do lead to a different bank response to CBDC competition. Under decreasing returns to scale, banks push back against the competition through higher deposit rates (and also lending rates in Appendix C.3). Instead, in the constant returns to scale setup, $r_d = A - \phi - 1$ and therefore the deposit rate is irresponsive to θ and r_{cbdc}

C.2 Anonymity externalities

In this extension, we consider the possibility that anonymous means of payment, like cash, are associated with negative externalities, due to the potential for illicit activities. There can be legitimate reasons that households desire anonymous forms of money, but by providing for that demand, the illicit uses of anonymity are also bolstered. In particular, we now let the utility of household h be given by

$$U_h(j) = \rho C_j - |x_j - h| - \eta_j - \beta \int_{n \neq h} x_{j(n)} dn \tag{57}$$

where $\beta \int_{n \neq h} x_{j(n)} dn$ captures the notion of negative externalities from anonymous means of payment. Here, $n \in [0,1]$ represents "all other households".[54] While every household with $h > 0$ likes anonymity in her own means of payments, every household also dislikes anonymity in other households' transactions. The weight $\beta \in [0,1]$ represents the extent to which the household dislikes others' anonymity in payment transactions.

Following the same steps as before, we derive unconstrained optimal policies as

$$\theta^{ce} = \frac{2 + \rho(2(A - \phi) - 1) - \beta(2 + \rho)}{4 + 3\rho - \beta(4 + \rho)} \tag{58}$$

$$r_{cbdc}^{ce} = -2\frac{\beta}{4 + \rho}\frac{(A - \phi)(4 + \rho) - 2(3 + \rho)}{4 + 3\rho - \beta(4 + \rho)} \tag{59}$$

which nest the solutions in (36) and (40) for $\beta = 0$.[55] The most interesting aspect of these solutions is that, for any $\beta > 0$, $r_{cbdc} \neq 0$ is now optimal, even when network effects play no role. Depending on parameter values, r_{cbdc}^{ce} can be either positive or negative. In particular, in relation to the value of bank intermediation, r_{cbdc}^{ce} moves inversely with θ^{ce}: A higher value of bank intermediation leads to a more cash-like optimal CBDC design and lower (including possibly negative) CBDC rates.

[54] Given that each individual agent is atomistic, the space of all agents excluding one agent remains defined on $[0, 1]$.

[55] The same holds for the *nce* solutions. These are not shown here in the interest of brevity, but are available on request.

This inverse relation between optimal CBDC rates and θ is intuitive, and derives from a ranking of forms of payment according to their anonymity externalities: cash is the worst, deposits are the best, and the CBDC is somewhere in between, depending on its design. When CBDC design is optimally quite similar to cash, then it is also optimal to have negative CBDC rates, to push more households into deposits, and limit the anonymity externalities induced by the CBDC. Instead, when the CBDC is more similar to deposits, then a positive CBDC rate is optimal, to help attract more households away from cash.

C.3 Bank market power

We now consider banks that compete à la Cournot in the loan market, taking the actions of other banks as given. Each bank therefore internalizes that total loans and the interest rates on those loans depend on its individual lending as follows

$$L = l + (1-\nu) L \rightarrow \frac{\partial L}{\partial l} = 1 \tag{60}$$

$$\frac{\partial R}{\partial l} = \frac{\partial R}{\partial L} = -1 \tag{61}$$

where $\frac{\partial R}{\partial L} = -1$ comes from equation (12). Here, ν represents the extent of bank market power, with the extremes of $\nu = 0$ and $\nu = 1$ representing, respectively, perfect competition (i.e., our baseline model) and a monopoly.

The bank's profit maximization problem is given by

$$\max_{l} \{(R(l) - r_d) l\} \tag{62}$$

where the bank recognizes the dependence of loan rates on an individual bank's lending decision: R depends on l. This yields the first-order condition

$$R(l) + \frac{\partial R(l)}{\partial l} l = r_d \tag{63}$$

Moreover, deposit market equilibrium is derived from $D = L$, where D is from s_d in (19):

$$L = \frac{\rho\left(r_d - r_{cbdc}\right) + \theta}{2} \tag{64}$$

Together, (12), (63), and (64) provide three equations in three unknowns, L, R and r_d. Replacing $\frac{\partial R}{\partial l} = -1$ from (61), and $l = \nu L$, we can solve this to obtain

$$L = \frac{\rho\left(A - 1 - r_{cbdc} - \phi\right) + \theta}{2 + \rho\left(1 + \nu\right)} \tag{65}$$

$$R = A - \phi - 1 - \frac{\rho\left(A - 1 - r_{cbdc} - \phi\right) + \theta}{2 + \rho\left(1 + \nu\right)} \tag{66}$$

$$r_d = \frac{2\left(A - 1 - \phi\right) + \left(1 + \nu\right)\left(\rho r_{cbdc} - \theta\right)}{2 + \rho\left(1 + \nu\right)} \tag{67}$$

Following the same steps as before, we again derive welfare and, from there, optimal policies

$$\theta^{ce} = \frac{8 + 2\rho\left(2\nu + \left(A - \phi\right)\left(4 + \rho + 2\rho\nu\right) - 1\right) - \rho^2(1 + (2 - \nu)\nu)}{16 + 3\rho^2\left(1 + \nu\right)^2 + 8\rho\left(2 + \nu\right)} \tag{68}$$

$$r_{cbdc}^{ce} = -2\nu\frac{1 + 3\rho\left(A - 1 - \phi\right)}{16 + 3\rho^2\left(1 + \nu\right)^2 + 8\rho\left(2 + \nu\right)} \tag{69}$$

where for $\nu = 0$ we retrieve our earlier solutions for optimal policies in (36) and (40). Indeed, by comparing the above expressions to (36) and (40), we can see the direction in which $\nu > 0$ pulls optimal policies. That is, using the expressions for θ^{ce} and r_{cbdc}^{ce} in (68) and (69), we numerically obtain that, over the parameter ranges in (17):

$$\inf \theta^{ce} - \theta^{ce}|_{\nu=0} = -\frac{279}{5372}, \ \sup \theta^{ce} - \theta^{ce}|_{\nu=0} = 0$$

$$\inf r_{cbdc}^{ce} - r_{cbdc}^{ce}|_{\nu=0} = -\frac{35}{163}, \ \sup r_{cbdc}^{ce} - r_{cbdc}^{ce}|_{\nu=0} = 0$$

and therefore $\nu > 0$ means that both θ^{ce} and r_{cbdc}^{ce} are lower than with $\nu = 0$. This emanates from the fact that greater market power in lending helps insulate banks from the negative impact of a CBDC. Although increased competition for retail funding still drives up banks' deposit rates, banks with market power partly compensate by also raising loan rates. In view of banks' increased ability to withstand the impact of a CBDC, the optimal CBDC design moves closer to deposits (lower θ), although the policy maker partly insulates the impact of this move by also cutting CBDC rates into negative territory.

C.4 Alternate equilibria under suboptimal policies

Table 1 listed three equilibria that do not occur under optimal policies. However, these equilibria can come about if policies are set suboptimally.

CBDC and cash Per Lemma 2, deposits never vanish under optimal policies. This is intuitive, since without deposits, our model yields zero intermediation, and the production of consumption goods shuts down. Nevertheless, it is easy to show that suboptimal policies could yield this equilibrium. For instance, for $\theta = 0$, if the CBDC rate is set such that

$$r_{cbdc} > A - \phi - 1 \tag{70}$$

then this ensures that $r_{cbdc} > r_d$ (by equation (23)), while the payment profile ($\theta = 0$) is equivalent to deposits. Hence, the CBDC strictly dominates deposits in this case: no household would choose to hold deposits.

CBDC only Any arbitrarily high r_{cbdc} would kill off both deposits and cash. Households would be paying for these CBDC interest payments through the lump-sum tax T, and therefore this scenario brings only disadvantages to households, who lose payment instrument variety and the productive benefits of bank intermediation, without gaining anything in return.

Cash and deposits There are three ways that a suboptimally designed CBDC could lead a situation where the design constraint (15) is violated such that there is no uptake of CBDC, and only cash and deposits are in use. First, CBDC could be designed in such a way that it is strictly dominated by cash, and violates (44). Second, CBDC design could imply that bank deposits are a strictly preferred form of payment, which occurs when (45) is violated. Third, even if the CBDC is not strictly dominated by cash or deposits, its design could be such that network effects prevent the buildup of a critical mass of CBDC users (15).

To give a concrete example, we replace r_d from (23) into (45). This yields

$$\left(\frac{1+\rho}{\rho}\right)\theta + r_{cbdc} > A - \phi - 1 \tag{71}$$

which means that when the policy combination (θ, r_{cbdc}) is set such that the condition above is violated, as for example for a sufficiently negative r_{cbdc}, deposits strictly dominate CBDC.

D Deriving a linear city of payment preferences

This appendix provides a stylized model highlighting how a linear-city model of payment preferences can be derived from microfoundations. The model is based on the notion that payment privacy can have value for households, when their digital transaction data can be used by private companies with monopoly power. We concentrate on a simple setup with cash and deposits only, and show how a "line" between these can arise endogenously, including a cutoff that determines household sorting. Once a spectrum of this sort is derived, formulating the intermediate case of a CBDC is a relatively straightforward extension.

In this model, deposit-based payments are processed by a fintech provider (or a bank that has a similar business model), which is capable of tracking all transactions and is legally unencumbered to use this data to its own benefit. The fintech company is also the sole provider of credit in the economy, and provides loans to households. Moreover, the only means that the fintech company has to assess the creditworthiness of its customers is by parsing their transactions data. For simplicity, we abstract from explicitly modeling deposit and lending markets and interest rates here, and instead focus purely on household choice based on the characteristics of deposits versus cash.

There are two types of products for households to purchase in this economy: G (Good) and B (Bad), where B can be considered a type of sin product, such as alcoholic beverages or cigarettes. Credit quality is inferred from the share of its income that a households spends on G. We assume identical incomes across households, and each household h determines what fraction $\gamma(h)$ to spend on good G. Each household has a preferred share of its income that it would like spend on each type of product: we denote by $p(h)$ the ideal fraction of household h's income spent on good G. Households are heterogeneous in their ideal consumption patterns. In particular, households are uniformly distributed on $p(h) \in [0, 1]$. Moreover, any distance between a household's ideal and actual consumption allocation, comes at a quadratic disutility cost to the household: $(\gamma(h) - p(h))^2$.

The key distinction between cash and deposits here, is that deposit transactions are monitored, while cash transactions are not. Monitoring matters because of the credit scores being assigned to households by the fintech company. For households using cash, the company cannot assign individualized credit scores, but rather uses an aggregate credit score, based on the consumption pattern of the average cash user. That is, all cash users are pooled together, in this respect. Instead, deposit using households are differentiated by the fintech company according to their own purchase behavior.

Importantly, once households use deposits for any fraction of their payments, they are unable to hide their overall purchase pattern from the fintech company. Endogenously, the model contains full revelation, because households have known, identical incomes.[56] If the fintech company observes a depositor using only a fraction $\gamma(h)$ of income, and fully using it on G, then the company infers that the household used the rest of its income to purchase B using cash. It is in this sense that deposits and cash cannot be effectively mixed: while the household is technically capable of mixing, the choice for using deposits at all, immediately implies full revelation: payment privacy is undiversifiable.

The aim of this appendix is purely qualitative, and as such we choose simple functional forms to highlight the relevant tradeoff. In particular, we let credit scores be a linear function of $\gamma(h)$ (for depositors) and assume that the utility derived from a higher credit score also enters linearly in the household's utility function. Household utility is given by

$$U(h) = \lambda E\left[\gamma(h) | j(h)\right] - \left(\gamma(h) - p(h)\right)^2 \tag{72}$$

where $j(h)$ is household h's chosen form of money, namely either d (deposits) or c (cash), λ is a parameter that weighs the utility value of the welfare score as compared to approximating

[56] More generally, the underlying assumption can be seen as a requirement on deposit-opening households to reveal their income to the fintech provider.

the household's ideal consumption shares, and

$$E\left[\gamma(h)\middle|\, j(h)\right] = \begin{cases} \gamma(h) \text{ if } j(h) = d \\ \widehat{\gamma} \text{ if } j(h) = c \end{cases} \tag{73}$$

where $\widehat{\gamma}$ equals the average share of G purchased by cash holders. Since households are atomistic, a given cash holder will always consume exactly the same as her bliss point: $\gamma(h) = p(h)$ when $j(h) = c$.

Instead, a depositor will solve the following optimization problem

$$\max_{\gamma(h)} \left\{\lambda\gamma(h) - (\gamma(h) - p(h))^2\right\} \tag{74}$$

leading to optimal consumption share of G

$$\gamma(h) = \frac{\lambda}{2} + p(h) \tag{75}$$

where $\frac{\lambda}{2}$ parameterizes the extent of overconsumption of G induced by monitored transactions.

The choice between cash and deposits then boils down to a comparison of utility under household optimal consumption. A household chooses deposits over cash if and only if utility as a depositor (setting $\gamma(h) = \frac{\lambda}{2} + p(h)$) is greater than utility as a cash holder (which equals $\lambda\widehat{\gamma}$). This becomes the following condition for choosing deposits:

$$\frac{\lambda}{4} + p(h) > \widehat{\gamma} \tag{76}$$

which can also be written as

$$p(h) > \widehat{\gamma} - \frac{\lambda}{4} = \overline{p} \tag{77}$$

This implies a sorting of households: households with $p(h) > \overline{p}$ choose deposits, while households with $p\left(h\right) < \overline{p}$ choose cash. That is, those households whose preferences favor a relatively large share of G consumption, are more eager to engage in a full revelation relationship with the fintech provider, in order to reap the benefits of an improved credit score. Instead, households with a relatively larger preference for consuming B, choose cash, opting out of a depositor relationship with the fintech provider that effectively "forces" them to overconsume G in order to appear more creditworthy. Overall, then, this model shows that heterogeneity in consumption preferences can translate into heterogeneous payment instrument choice.

E Privately issued digital currency

Our model centers on the issuance of a digital currency by a central bank. This extension considers how the model's outcomes would change if the digital currency were instead issued by a private company. We refer to the private digital currency as PDC and compare its privately optimal design to the welfare optimal design of a CBDC.

The private issuer's optimization problem is given by

$$\max_{\theta, r_{pdc}} \left\{ \sigma \left(1 - \theta^2\right) s_{pdc} - r_{pdc} s_{pdc} \right\} \tag{78}$$

The private issuer has two parts to its objective function. First, it cares about interest income or cost. The term $-r_{pdc}s_{pdc}$ represents this, where r_{pdc} is the interest paid (or received if negative) on the PDC and s_{pdc} is the share of households opting for the PDC as their payment instrument.[57] Second, the issuer cares about the acquisition of payment data from the use of its digital currency, which can be of value in targeting other products or services to households, as explored for the case of credit provision in Appendix D.[58] This is represented by the term $\left(1 - \theta^2\right) s_{pdc}$: data acquisition is proportional to the size of the digital currency's user base, s_{pdc}, and is also related to the anonymity of the digital currency by $\left(1 - \theta^2\right)$. A fully anonymous digital currency with $\theta = 1$ would not allow the issuer to obtain household specific payment data. As the currency moves away from full anonymity (θ declines from 1), it quickly becomes easier to obtain more household relevant payment data, as captured by the quadratic form. Lastly, σ represents the weight that the company places on obtaining payment data relative to interest income.[59]

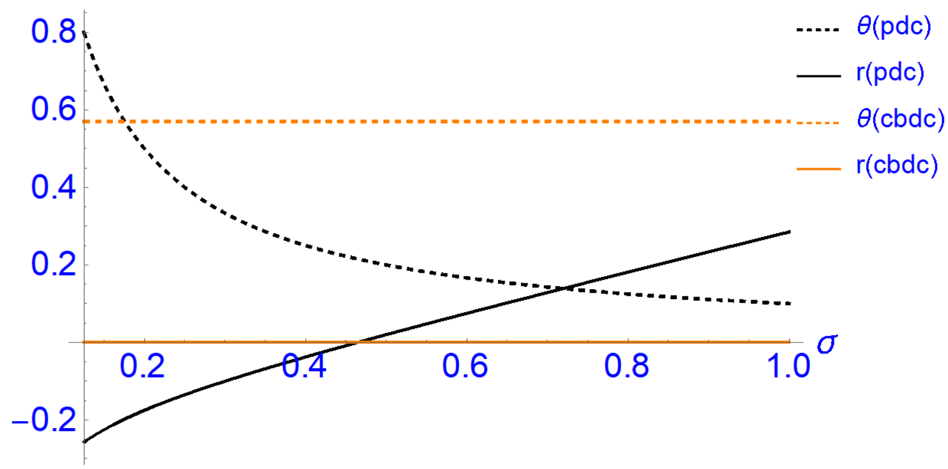
In (78), s_{pdc} is given by the same closed form expression obtained for s_{cbdc} in Lemma 3,

[57] For simplicity, we abstract from a fixed revenue that the issuer would make from investing the receipts from the PDC.

[58] The PDC issuer does not necessarily need to operate in sectors which value data, as it could also extract value from the data by selling it to third parties.

[59] In addition, to the parameter conditions in (17), we now also set $\sigma \geq \frac{1}{8}$, which is a sufficient condition for deriving an interior ($\theta \in [0, 1]$) solution to this optimization problem. The derivation of this sufficient condition, as well as other calculations in this Appendix, are contained in the *Mathematica* file that is available on request.

Figure 5: **Optimal PDC vs. optimal CBDC**



Note: When the PDC issuer cares only about interest income, it sets a negative PDC interest rate and makes the PDC more cash-like than socially optimal. Instead, if the PDC issuer places emphasis on payment data acquisition, it offers positive CBDC interest and the PDC competes more than socially optimal with deposits.

except that r_{cbdc} is replaced with r_{pdc}:

$$s_{pdc} = \frac{2 + \rho\left(3 - 2\left(A - \phi\right) + \left(4 + \rho\right) r_{pdc} + \theta\right)}{2\left(2 + \rho\right)} \tag{79}$$

We can now solve the optimization problem in (78). The PDC issuer sets:

$$\theta_{pdc} = \frac{1}{\left(8 + 2\rho\right)\sigma} \tag{80}$$

$$r_{pdc} = \frac{1}{8}\left(4\sigma - \frac{2}{\rho} + \frac{2\left(4\left(A - \phi\right) - 5\right)}{4 + \rho} - \frac{3}{\left(4 + \rho\right)^2 \sigma}\right) \tag{81}$$

To visualize the difference between the privately optimal policies of the PDC issuer in (80) and (81), as compared to the socially optimal policies of the central bank in (36) and (40), Figure 5 plots the optimal policies relative to σ for a numerical example (namely, $\rho = 1$ and $(A - \phi) = \frac{3}{2}$).

When σ is low, the PDC issuer puts little value on data acquisition and instead concentrates on maximizing interest revenue. It therefore sets a negative PDC interest rate. It also shies away from competing with deposits that offer a positive interest rate, and introduces a relatively cash-like PDC. Competing with bank deposits has both costs and benefits for a PDC issuer: the cost is that, unlike cash, deposits pay positive rates and these rates rise further when competition intensifies; the benefit is that positioning the PDC closer to deposits allows the issuer to extract more useful payment data. This benefit is limited when σ is low.

Instead, when σ is high, the PDC issuer centers attention on maximizing data acquisition. This leads it to compete with bank deposits, both because a less anonymous PDC is now more valuable to the issuer and because deposits have a larger share of users than cash (due to their positive interest rates) and increasing PDC user share is key for obtaining payment data. The PDC issuer takes a loss on its interest income, offering a positive PDC interest rate that allows it to compete with bank deposits.

How the PDC issuer's optimal policies diverge from socially optimal policies, depends on whether σ is low or high. When σ is low, the PDC is more cash-like than socially optimal, because the PDC issuer focuses on maximizing interest income and neglects the social benefits of increased diversity among payment instruments. When σ is high, the PDC is instead more deposit-like than socially optimal, and the PDC issuer's focus on data acquisition imposes negative externalities on bank intermediation.

We also note that, while there is a value of σ where $\theta_{pdc} = \theta_{cbdc}^{ce}$ and a value of σ where $r_{pdc} = r_{cbdc}^{ce}$, these occur at different values of σ. That is, optimal PDC design never coincides with welfare-maximizing CBDC design.

The properties shown in Figure 5 are not specific to the chosen numerical example:

$$\frac{\partial \theta_{pdc}}{\partial \sigma} = -\frac{1}{2\left(4+\rho\right)\sigma^2} \tag{82}$$

$$\frac{\partial r_{pdc}}{\partial \sigma} = \frac{1}{8}\left(4+\frac{3}{\left(4+\rho\right)^2\sigma^2}\right) \tag{83}$$

mean that $\frac{\partial \theta_{pdc}}{\partial \sigma} < 0$ and $\frac{\partial r_{pdc}}{\partial \sigma} > 0$ are true for any ρ and σ. We can also show that, at the lower bound of σ, θ_{pdc} is always more than "halfway" to cash. At $\sigma = \frac{1}{8}$, we have that

$\theta = \frac{1}{1+\frac{1}{4}\rho}$ and since $\frac{3}{4} \leq \rho \leq \frac{3}{2}$ from (17), we know that the lowest value θ_{pdc} can take at $\sigma = \frac{1}{8}$ is

$$\inf_{\sigma=\frac{1}{8}} \theta_{pdc} = \frac{8}{11} > \frac{1}{2} \tag{84}$$

Adding considerations of network effects, would amplify the divergence between the privately optimal PDC and socially optimal CBDC policies. In this extension, we have focused on the case without network effects, where all three forms of money (the third being either PDC or CBDC) are in use. Bringing in network effects, would give the PDC issuer additional incentives for socially adverse behavior. Instead of the central bank's desire to sustain different forms of payment where possible, the PDC issuer would rather eliminate competing forms of payment.