Wholesale Bank Funding, Capital Requirements and Credit Rationing

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Abstract

This paper analyzes how different types of bank funding affect the extent to which banks ration credit to borrowers, and the impact that capital requirements have on that rationing. Using an extension of the standard Stiglitz-Weiss model of credit rationing, unsecured wholesale finance is shown to amplify the credit market impact of capital requirements as compared to funding by retail depositors. Unsecured finance surged in the pre-crisis years, but is increasingly replaced by secured funding. The collateralization of wholesale funding is found to expand the extent of credit rationing. As robustness, banks with mixed funding sources are also considered.

Keywords: Rationing, Capital requirements, Regulation, Wholesale finance, Deposit Insurance

JEL Classification: G21, G28
1 Introduction

The policy discussions surrounding the new Basel Accords have largely centered around the question to what extent higher capital requirements will affect the availability of bank credit to borrowers, and thereby the real economy, with banks generally arguing the impact will be large, and regulators saying the opposite (BIS, 2010; IIF, 2011). This paper aims to enrich this debate by adding a new dimension to it: the mode of bank finance.

The manner in which banks fund themselves has undergone quite dramatic changes during the past decade. First, in the years before the crisis, unsecured wholesale bank finance boomed (Brunnermeier et al., 2009; Diamond and Rajan, 2009), and the fraction of traditional, insured retail deposits among bank liabilities declined. Banks, especially the largest ones, could generally obtain large volumes of short-maturity debt from wholesale financiers at low cost. Then, as the financial crisis unfolded, wholesale bank funding fell sharply, and has not fully recovered since. Importantly, moreover, the mode of wholesale finance has undergone a transformation, as banks increasingly turn to short-term debt contracts that are collateralized. This so-called "covered bond" or secured wholesale finance now constitutes a sizeable portion of bank funding. Figure 1 below depicts the development of wholesale bank funding in the euro zone.

In this figure the blue bars represent unsecured wholesale funding to eurozone banks, which grew rapidly between 2002 and 2006. The combined yellow and green bars are secured funding (where retained bonds are those that are retained by a bank rather than sold to investors, usually as a means to obtain repo financing from the ECB when needed). Since 2007 the share of such collateralized funding in overall wholesale bank funding has increased considerably.

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1The debate on the credit effects of capital requirements has a long history in the economic literature, mainly dating back to the the early 1990s credit crunch, with evidence indicating that the imposition of Basel I capital requirements exacerbated this episode. See Bernanke et al. (1991), Furlong (1992), Jacklin (1993), Haubrich and Wachtel (1993), Hancock and Wilcox (1994), Hancock et al. (1995) and Peek and Rosengren (1995a,b). For an opposing view, see Berger and Udell (1994).

2This figure is taken from FT article "Fitch on a brave new (bank funding) world", June 16th, 2011. The sources used by the FT are Fitch and Deallogic.
Our paper starts by extending the seminal model on credit rationing, Stiglitz and Weiss (1981) (henceforth SW), to allow for bank default, which makes it possible to differentiate between debt and equity financing. In SW rationing comes about through the volatility of borrower’s returns. Borrowers are indistinguishable to banks, and higher risk borrowers are willing to accept higher loan rates. A bank’s loan rate then has a sorting effect: the higher the rate, the greater the volatility of returns among the pool of loan applicants. This can lead banks to optimally charge a loan rate below the market clearing rate, which implies excess demand for loans in equilibrium.

When the regulator raises capital requirements banks are forced to delever their balance sheets. This expands credit rationing in two ways. Firstly, smaller balance sheets reduce the amount of credit that banks can supply. And, secondly, with a lower debt-to-equity ratio banks have a greater incentive to reduce the volatility of their asset portfolio. They are willing to give up more returns on successful loans in order to improve on the composition of loan applicants. Therefore, they lower loan rates, which increases credit demand. With fewer loans supplied and more demanded, credit rationing rises.\(^3\)

\(^3\)It may be seen as surprising that higher capital requirements lower bank’s optimal loan rates in this model. This comes from an incentive effect, as opposed to a cost-price effect, which can be found in Thakor’s (1996) model, discussed below. Empirically, in fact, the effect of higher capital on bank loan rates is a debated issue. Hubbard et al. (2002), Santos and Whinton (2010) and Lown and Peristiani (1996) find that better capitalized banks charge lower loan rates, whereas Fisher et al. (2010) report the opposite. Note, however, that these findings concern "voluntarily" raised...
We next introduce different types of bank debt. We consider in turn deposit-insured retail funding, unsecured wholesale funding and collateralized wholesale funding, in each case assuming that the bank’s debt consists entirely of that one given type of funding. Our first main result is that with unsecured wholesale funding the impact of capital requirements is larger than with retail funding. That is, although higher capital requirements always cause a rise in the incidence of credit rationing, this effect is greater when banks are funded at wholesale. The reason is that when banks are wholesale funded their funding rates and loan rates interact. Insured depositors do not care about how risky a bank is, because they are certain to get back their money if it fails. But unsecured financiers demand higher funding rates when a bank pursues a riskier (higher loan rate) strategy. When due to higher capital requirements a bank internalizes more of its risk, and reduces loan rates, its wholesale funding rates fall, which further raises the bank’s charter value, and make it more risk-averse, amplifying the effect on credit rationing.

Our second main result is that when banks move from unsecured to secured wholesale funding, more borrowers get credit rationed. The reason is, firstly, that banks reduce lending to borrowers as they need to place more of their portfolio in safe collateral. Secondly, there is a direct effect on the bank’s risk taking incentives: the collateral that it puts up makes it more averse to adverse selection problems. And thirdly, there is an indirect effect that runs through the financiers, who demand lower funding rates when covered by collateral, which interacts with bank loan rates in the manner described before.

Of course, generally a bank is not funded using only one type of funding. Rather, it accesses various forms of funding simultaneously. In an extension we analyze mixed finance banks, focussing on retail and unsecured wholesale. We consider two different setups, one in which there is a single representative mixed financier, and another in which there is a simple simultaneous moves game between a retail financier and an unsecured wholesale financier. We show that the game has the same outcome as the representative mixed financier, an outcome that is a direct extension of the results on one-type finance banking, namely that a larger share of unsecured finance amplifies capital rather than regulatory bounds. The distinction between voluntary and mandatory capital holding is formalized neither in my paper nor in Thakor’s (1996).
the impact of capital requirements.

It is important to note that the analysis in this paper is purely positive, not normative. We do not take a stance on the welfare effects of credit rationing, and instead purely focus on how the extent of rationing - the gap between credit demand and credit supply - is affected. Hence, we do not draw policy implications, that is, we do not say that some form of bank financing is better or worse because of its effects on credit rationing. Rather, the aim of the paper is to help foster an understanding of how credit market reactions to capital requirements differ according to bank financing forms.

The closest relative to this paper is that of Thakor (1996), who develops a borrower screening mechanism through which capital requirements are related to credit rationing. In his model banks choose the probability with which they screen a loan applicant. When capital requirements rise, banks face a higher cost of loan-funding. In response, all banks reduce the probability with which they screen applicants and thus more potential borrowers get rationed. Thakor (1996) does not consider different bank funding modes.

The next section introduces the basic model of credit rationing. Sections 3, 4 and 5 consider retail, unsecured wholesale and secured wholesale finance, respectively. Section 6 analyzes mixed funding. Finally, section 7 concludes.

2 Model

The basic setup follows SW, until the introduction of limited liability for banks (equation (7) onwards). We discuss SW’s model only briefly here, and refer to their paper for a more detailed exposition.

A Bank assets

In this model both borrowers and banks are risk neutral. There is a discrete number, \( N \), of projects, each of which is tied to one given borrower. Projects are numbered \( \theta = \{1, 2, \ldots, N\} \), and the return
on project $\theta$ is called $R_\theta$. It is known that all projects have the same mean return, but they differ in the volatility of returns. The distributions of projects’ returns, $F(R_\theta)$, are known to borrowers but not to banks. Borrowers’ projects are sorted by their risk, where a larger $\theta$ corresponds a riskier borrower (in terms of a mean-preserving spread). By the definition of second-order stochastic dominance:

$$\int_0^\infty R_2 f(R_2) dR = \int_0^\infty R_1 f(R_1) dR$$

and for $y \geq 0$

$$\int_0^y F(R_2) dR \geq \int_0^y F(R_1) dR$$

To start up a project, a borrower needs an amount $B$, which he can obtain at the prevailing bank loan rate, $b_r$. If the return on his project is insufficient to pay the bank back the promised amount, he defaults on his loan. Formally, this occurs when:

$$C + R_\theta \leq (1 + \bar{r}^b) B$$

where $C$ is the collateral pledged on the loan. Then, the net return to a borrower can be written as:

$$\max \{ R_\theta - (1 + \bar{r}^b) B, -C \}$$

By SW Theorem 1, for a given loan rate, $\bar{r}^b$, there is a critical value $\hat{\theta}$ such that a firm borrows from the bank if and only if $\theta \geq \hat{\theta}$. Thus, $\theta \geq \hat{\theta}$ constitutes the set of individuals that requests loans from the bank. With high returns borrowers make large profits, with low returns they default. Therefore, greater volatility implies greater expected returns, and a willingness to pay higher loan rates. Then, by SW Theorem 2: $\frac{d\bar{\theta}}{d\sigma} > 0$. A higher loan rate brings about a riskier pool of loan applicants.

On each borrower to which the bank lends it receives

$$\min \{ R_\theta + C, (1 + \bar{r}^b) B \}$$
and the bank’s total revenues can be written as

\[ \sum_{\theta \in \Theta} \min \{ R_{\theta} + C, (1 + \hat{r}) B \} \] (6)

where \( \Theta \) is the subset of loan applicants \( \theta \geq \hat{\theta} \) to which the bank is randomly matched.

**B  Bank liabilities**

The bank’s funding consists of equity, \( Q \), and debt, \( D \):

\[ X = Q + D \] (7)

Its only costs are the payments to its creditors \( (1 + \hat{r}^d) D \), where \( \hat{r}^d \) is the return that creditors demand on the funds they provide. This funding rate is exogenous now, but is endogenized in subsequent sections. We assume that a bank has a given amount of equity, \( Q = \bar{Q} \), and that any required adjustments to its balance sheet are made by adding or reducing debt, rather than by issuing or shedding equity. In particular, like Thakor (1996), we assume that capital requirements are always binding. Implicitly equity is assumed to be a more expensive form of financing than debt, and banks prefer to take on as much debt as is allowed. Capital requirements are defined here as fixing a minimum to the ratio \( \frac{Q}{X} \): at least a given fraction of liabilities must be in equity. For given \( \bar{Q} \) this means that the regulator’s capital requirements fixes a maximum debt level (called \( D^{\text{max}} \)).

\[ ^4 \text{We thus focus on unweighted capital requirements. These are similar to the leverage ratio caps in Basel III.} \]
### Bank profits

If revenues exceed costs the residual is paid out to shareholders. If not, then the bank defaults. Total profits can then be written as

$$
\Pi = \max \left\{ \left( \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}_\theta) B \} \right) - (1 + \tilde{r}_d) D, 0 \right\}
$$

where the bank’s problem is

$$
\max_{\tilde{r}} E [\Pi]
$$

Note that SW, beyond stating that banks are not price takers (p.395), do not discuss the mode of competition between banks. There is no need to choose a specific type of competition here either. We do assume that banks are identical (meaning here that they all have the same amount of equity, $Q$).

### Comparative statics to credit rationing

Define the extent of credit rationing as

$$
\Omega = B \left( N - \tilde{\theta} \right) - MX
$$

where $B \left( N - \tilde{\theta} \right)$ is total loan demand by borrowers, and, letting $M$ be the number of banks, $MX$ is total loan supply.

**Lemma 1** Higher capital requirements imply more credit rationing: $\frac{\partial \Omega}{\partial (Q/X)} > 0$.

**Proof.** In the appendix. ■

Lemma 1 shows that credit rationing increases in capital requirements. Intuitively, equity holders only receive what remains of returns after bank creditors have been paid. They are like the owners of a call option. The larger the debt burden of a bank, the more its shareholders have to gain from upside risk. And the less they have to lose from downside risk, since the bank can default.
when returns are low. If bank management represents shareholders’ interests, therefore, it will like volatile returns more when the bank is more debt financed. Forcing the bank to shed leverage implies that the taste for volatility declines. The bank will charge a lower loan rate to improve on the quality of its borrower pool. At lower loan rates demand for bank credit rises, while at the same time banks supply less credit due to their downsized balance sheets. Thus, credit rationing becomes more prevalent.

3 Retail funding

So far, we have assumed a constant, exogenous bank funding rate, $\bar{r}^d$. Now instead we will consider bank financiers as explicit agents, who are risk neutral and active on a perfectly competitive financial market that drives financiers’ expected excess returns to zero. In particular, we model a simultaneous moves game between a representative bank and a representative financier, whose decision variables are the loan rate and the funding rate, $\bar{r}^b$ and $\bar{r}^d$, respectively. As can be seen from equations (8) and (9), the bank’s decision will depend upon the funding rate. And, in general, the financier’s decision will depend upon the bank’s loan rate, so that the equilibrium is determined by the intersection of two reaction functions.

We start from a simpler case, however, in which the financier’s decision is not affected by that of the bank, because of fully insured deposits. We assume an exogenously funded deposit insurance that provides 100% coverage to bank creditors. This represents the case of retail depositors, who are usually protected by full insurance for at least a certain portion of their deposits. For simplicity we set the risk-free rate to zero, so that, trivially,

$$ (\bar{r}^d)^M = 0 \quad (11) $$

where $(\bar{r}^d)^M$ is the equilibrium market funding rate.

Next, we turn to the bank’s reaction function. Let $(\bar{r}^b)^*$ denote the bank’s optimal loan rate to borrowers. Then,
Lemma 2 \( \frac{\partial (\hat{r}^b)^*}{\partial r^d} > 0 \).

**Proof.** In the appendix. ■

This means that the bank’s reaction function is positively sloped: given an increase in the funding rate the bank optimally raises its loan rate. We can graphically depict the bank’s reaction function, and use it to visualize the impact of an increase in capital requirements. This is shown in figure 2, where the financier’s funding rate is on the horizontal axis and the bank’s loan rate on the vertical axis.

![Figure 2: capital requirements’ impact if retail funded](image)

The dashed line is the financier’s reaction function: the funding rate \( (\hat{r}^d)^M \) that he charges given the bank’s risk profile embodied in the bank’s loan rate rate, \( \hat{r}^b \). Here, this reaction function is just a vertical line at zero, since deposit insurance absolves the financier of all funding risk. The solid black lines represent the bank’s reaction function given low and high capital requirements, respectively. The positive slope, formally derived in Lemma 2, results from the fact that higher funding costs make a bank more indebted, lowering its charter value, and raising its risk-taking incentives. Higher risk translates into a higher loan rate to borrowers, as the bank is less concerned about the volatility in its returns induced by adverse selection (i.e., lending to a riskier pool of
The impact of higher capital requirements is precisely opposite: with higher capital a bank internalizes more of its borrowers’ risk, so that for any given funding rate \( \tilde{r}^d \) its optimal loan rate \( (\tilde{r}^b)^* \) is lower. Therefore, the reaction function shifts downward, as shown in figure 2.

4 Unsecured wholesale funding

In this section we consider bank financiers who are not covered by any type of explicit or implicit guarantees, nor do they receive any collateral on their loans. This is taken to represent the case of the unsecured wholesale funding market.\(^6\) Thus, they care about bank risk. In particular, the financiers’ net expected return is given by

\[
E \left[ \min \left\{ \sum_{b \in \Theta} \min \left\{ R_b + C, (1 + \tilde{r}^b) B \right\} - D, \tilde{r}^d D \right\} \right]
\]

which means that the return is \( \tilde{r}^d D \) if the bank repays in full, and is the residual value of the bank minus the lost principal if the bank cannot repay. In this case the market funding rate is given by the solution to

\[
(\tilde{r}^d)^M = \tilde{r}^d : 0 = E \left[ \min \left\{ \sum_{b \in \Theta} \min \left\{ R_b + C, (1 + \tilde{r}^b) B \right\} - D, \tilde{r}^d D \right\} \right]
\]

from which we can derive that

\textbf{Lemma 3} With unsecured wholesale funding \( \frac{\partial (\tilde{r}^d)^M}{\partial \tilde{r}^b} > 0. \)

\textbf{Proof.} In the appendix. \( \blacksquare \)

\(^5\)The relationship between bank funding rates and bank risk taking is also interesting from the perspective of the literature on the “risk-taking channel of monetary policy”, in which lower policy rates can lead to the buildup of financial imbalances (Borio and Zhu, 2008; and Adrian and Shin, 2009). Empirical work testing this channel includes, among others, Altunbas et al. (2010) and Maddaloni and Peydró (2011).

\(^6\)Recent experience may place in doubt the assumption that unsecured wholesale financiers are not covered by some implicit government guarantees. The case of partial deposit insurance is investigated in section 6A.
We can use this property to derive our main result on the effects of unsecured wholesale funding. Let us call the derivative in Lemma 1 \( \frac{\partial \Omega}{\partial (Q/X)} \) the "impact of capital requirements", that is, the extent to which capital requirements affect credit rationing. And let the superscript "UF" stand for unsecured funding and "DI" for retail funding with deposit insurance. Then,

**Proposition 1** The impact of capital requirements is larger with unsecured wholesale funding than with retail funding:

\[
\left[ \frac{\partial \Omega}{\partial (Q/X)} \right]^{UF} > \left[ \frac{\partial \Omega}{\partial (Q/X)} \right]^{DI}.
\]

**Proof.** In the appendix. ■

The intuition for this result can be seen from a comparison between figures 2 and 3, where figure 3 represents the case of a change in capital requirements when the bank is financed with unsecured wholesale debt. As can be seen from the length of the arrow along the vertical axis, capital requirements have a greater effect on loan rates when the bank is wholesale funded than when it is retail funded. In the SW model a reduction in bank loan rates is equivalent to a decline in risk appetite (i.e. more internalization of the volatility of borrower returns), which translates into a larger gap between demanded and supplied credit, and hence more rejected loan applicants. Thus, a greater reduction in loan rates when wholesale funded means a larger impact on the extent of credit rationing.

The reason that unsecured wholesale finance amplifies the impact of capital requirements is the feedback relation that it induces between funding rates and loan rates. When financiers are uninsured and care about bank risk, any change in bank loan rates, which alters the riskiness of the bank’s borrowers, is reflected in bank funding rates. But bank funding rates feed back into the bank’s optimal loan rate. The effects are mutually reinforcing, as both derivatives are positive: higher loan rates imply higher risk and therefore higher funding rates; higher funding rates imply a lower bank charter value, more risk taking incentives, and thereby higher optimal loan rates.
Secured wholesale funding

We now consider collateralized bank funding, which means that a bank has to give collateral, which its financier can retain in case the bank is unable to fulfil its obligations. We assume that on its total funding $D$, the bank has to give collateral worth $\Gamma < D$. This collateral must be in high quality - risk free - instruments (and, like borrower collateral in the basic SW model, the amount of desired collateral is not linked to the riskiness of the bank). Thus, the bank invests a fraction $\frac{\Gamma}{X}$ of its balance sheet in a risk-free asset, while the rest $(\frac{X-\Gamma}{X})$ goes to loans to borrowers. The funding market rate is now given by:

$$ (\hat{r}^d)^M = \hat{r}^d : 0 = E \left[ \min \left\{ \sum_{\theta \in \Theta} \min \left\{ R_0 + C, (1 + \hat{r}^b) B \right\} - D + \Gamma, \hat{r}^d D \right\} \right] $$

(14)

Lemma 4 $\frac{\partial \hat{r}^d}{\partial \Gamma} < 0$.

Proof. In the appendix. ■
It is quite intuitive that when a bank puts up more collateral, its funding costs decline, as financing becomes less risky to financiers. Apart from the effect on the financier’s reaction function, however, the collateralization of funding also affects the bank’s reaction function. In particular, the bank’s optimization problem becomes

\[
\max_{\hat{p}^h} \mathbb{E} \left[ \max \left\{ \sum_{\theta \in \Theta} \min \left\{ R_{\theta} + C, (1 + \hat{p}^h) B \right\} - (1 + \hat{p}^d) D, -\Gamma \right\} \right] \tag{15}
\]

**Lemma 5** \( \frac{\partial \mu^h}{\partial \mu} < 0 \).

**Proof.** In the appendix. ■

The impact of increased collateralization on the bank’s reaction function, can be interpreted through the bank’s altered limited liability. Defaulting leads to a loss of collateral, and hence there is more at stake for the bank: it optimally internalizes more of the volatility of borrower returns. This decline in bank risk appetite leads to increased rationing, as the bank aims to mitigate adverse selection problems by lowering the loan rate.

**Proposition 2** *Increasing the collateralization of wholesale funding expands credit rationing:* \( \frac{\partial \Omega}{\partial \mu} > 0 \).

**Proof.** In the appendix. ■

We depict this result graphically in figure 4. Essentially, there are three channels through which higher bank collateral affects the extent of credit rationing. The first channel is that as banks are forced to raise risk-free collateral, they cut back on loans to borrowers, thereby reducing credit supply. The second channel is that because banks internalize more of the risk they take, they cut loan rates to improve the pool of borrowers (the bank reaction function shifts downward). The third channel is that financiers reduce funding rates because their risks are partly covered by the collateral (the financier reaction function shifts to the left). And because the lower funding rates raise bank charter values, bank’s optimal risk taking is further reduced, amplifying the fall in the loan rates. Thus, the pool of borrowers expands while the supply of credit contracts, and rationing
becomes more prevalent.

![Figure 4: Impact of collateralizing bank funding](image)

### 6 Mixed funding

Banks do not source from one type of financier only. Rather they have mixed sources of funding. This section considers a bank that is funded in part with retail deposits and in part with wholesale funding. We focus on unsecured wholesale funding here.

There are two ways that we could incorporate mixed funding within our model. The first method is to have a single, representative mixed financier. That is, all financiers hold equal shares of their credit to the bank in retail and wholesale deposits, and this is represented by a single representative mixed financier. The second method is to have a separate retail depositor and a wholesale financier as part of a multi-player game. We explore both options.
A Representative mixed financier

We assume that the financier has a share $\alpha < 1$ in insured (retail) deposits and $1 - \alpha$ in unsecured wholesale funding. Then, the funding market rate is given by:

$$(\hat{r}^d)^M = \hat{r}^d : 0 = E \left[ \min \left\{ \alpha \hat{r}^d D + (1 - \alpha) \left[ \sum_{\theta \in \Theta} \min \left\{ R_\theta + C, (1 + \hat{r}^b) B \right\} - D \right], \hat{r}^d D \right\} \right]$$

$$= \alpha \hat{r}^d D + (1 - \alpha) E \left[ \min \left\{ \sum_{\theta \in \Theta} \min \left\{ R_\theta + C, (1 + \hat{r}^b) B \right\} - D, \hat{r}^d D \right\} \right]$$

(16)

**Proposition 3** The larger the share of insured deposits, the greater the extent of credit rationing:

$$\frac{\partial \alpha}{\partial \alpha} > 0.$$

**Proof.** In the appendix.

When a larger fraction of deposits are insured, a bank can fund itself more cheaply. This raises its charter value and makes it more averse to borrower volatility. Its reduced risk appetite results in more conservative lending behavior - lower loan rates - and an expanded incidence of rationing.

We wish to caution, however, that this result must be taken with a grain of salt. Generally, deposit insurance is thought to raise bank risk taking incentives, because it reduces depositor monitoring: when depositors have less at stake they invest less in monitoring bank activities, which leads to moral hazard on the bank’s part. In our model there is no monitoring decision, and depositors always know as much (or, rather, as little) about the bank’s borrowers as the bank itself does.

**Proposition 4** The larger the share of insured deposits, the smaller the impact of capital requirements:

$$\frac{\partial^2 \Omega}{\partial (\frac{Q}{X}) \partial \alpha} < 0.$$

**Proof.** In the appendix.

This result arises from the fact that financiers are less concerned about bank risk when they are covered by more deposit insurance, which dampens the feedback between loan rates and funding rates, as discussed below Proposition 1. The overall effect of an increase in $\alpha$ is shown in figure
5. The financier’s reaction function shifts leftward (Proposition 3) and becomes more positively sloped. The latter means that when the bank’s reaction function shifts down due to a rise in capital requirements, the impact on loan rates, and thereby on credit rationing, is more subdued (Proposition 4). This can be seen in figure 5 from the fact that the vertical distance between the red dots is smaller after \( \alpha \) has risen (i.e., the financier reaction function has pivoted inwards).

![Figure 5: mixed funding](image)

B Multi-player game

We now consider the case in which there are distinct retail and wholesale financiers (each representing a continuum of its type). As before, a given fraction, \( \alpha \), of the bank’s funding comes from retail and \( (1 - \alpha) \) from wholesale. The game consists of two stages. In the first stage each financier type tells the bank his offer curve: what funding rate he demands for a given bank loan rate. In the second stage, the bank takes these curves as given, and sets its optimal loan rate. We assume that if the bank fails, the share of the remaining bank value that is given to each financier is proportional to the share of funding.

What makes the game relatively simple to solve is that the retail depositor’s behavior is inde-
pendent of that of the wholesale financier: like in figure 2 it has a vertical reaction function at the risk-free rate. The funding rate demanded by the wholesale financier is then the solution to

\[
(\hat{r}^d)^M = \hat{r}^d;
\]

\[
0 = E \left[ \min \left\{ \left( 1 - \alpha \right) \sum_{\theta \in \Theta} \min \left\{ R_\theta + C, \left( 1 + \hat{r}^b \right) B \right\} - (1 - \alpha) D, (1 - \alpha) \hat{r}^d D \right\} \right]
\]

\[
= E \left[ \min \left\{ \sum_{\theta \in \Theta} \min \left\{ R_\theta + C, \left( 1 + \hat{r}^b \right) B \right\} - D, \hat{r}^d D \right\} \right] \quad (17)
\]

which is equivalent to equation (13). From this we can show that:

**Proposition 5** The outcome, in terms of \((\hat{r}^b)^*\) and \(\Omega\), to the multi-player game is identical to that with the representative mixed financier.

**Proof.** In the appendix. ■

And, hence, the single mixed financier is representative of the game between a separate retail and wholesale financier.

### 7 Conclusions

Bank funding has gone through a roller coaster ride over the past decade. In the years before the crisis the broadening of wholesale markets, together with a stable, low interest rate climate, allowed banks to tap into new sources of funds, mainly at short maturities. As funds dried up during the financial crisis, and investor appetite for exposure to unsecured bank debt has remained subdued since, banks have increasingly resorted to collateralization to furnish their funding needs.

This paper has considered one aspect that changing bank funding patterns may impact upon, namely the provision of credit to bank borrowers. It has coupled this in particular to the impact of capital requirements, as the interaction between the rising regulatory burden on banks and the credit market has been at the center of policy debates. The key findings, that wholesale funding raises the impact of capital requirements as compared to retail funding, and that the collateralization of...
loans to banks can expand credit rationing, do not offer any direct guidance to policy makers, since these have not been coupled to the macroeconomy or welfare. They may, however, raise awareness that links between bank funding modes, capital requirements and the credit market exist, possibly enriching the policy debate.
Appendix: Proofs

Proof of Lemma 1. Within the bank’s profit in equation (8), consider first the revenue term given by (6). By the arguments in SW, this term is first increasing and then decreasing in $\bar{r}_b$. The reason is that beyond a threshold the negative sorting effect, through $\frac{\partial \theta}{\partial \bar{r}_b}$, dominates the higher return per successful loan $(1 + \bar{r}_b)$. Define $\bar{r}_b$ as this threshold:

$$\bar{r}_b \equiv \bar{r}_b : \frac{\partial}{\partial \bar{r}_b} \sum_{\theta \in \Theta} \min\left\{ R_{\theta} + C, (1 + \bar{r}_b) B \right\} = 0$$

and call the bank’s optimal loan rate $(\bar{r}_b)^\ast$. We first show that $(\bar{r}_b)^\ast \geq \bar{r}_b$ and then consider the trade-off of bank maximization that takes place for such values.

The reason that $(\bar{r}_b)^\ast \geq \bar{r}_b$ is that the option value created by the bank’s limited liability is affected by the volatility of its returns. In particular, we can rewrite (9) to

$$\max_{\bar{r}_b} E \left[ \max \left\{ \sum_{\theta \in \Theta} \min\left\{ R_{\theta} + C, (1 + \bar{r}_b) B \right\} - (1 + \bar{r}_d) D, 0 \right\} \right] = \max_{\bar{r}_b} E \left[ \max \left\{ \sum_{\theta \in \Theta} \min\left\{ R_{\theta} + C, (1 + \bar{r}_b) B \right\}, (1 + \bar{r}_d) D \right\} - (1 + \bar{r}_d) D \right] (19)$$

which has the structure of a call option with expected payoff $E [\max \{y, k\}]$ where $y$ is the value of the underlying asset and $k$ is the option’s strike price. By the standard arguments of option theory, the expected value of a call option is increasing in the volatility of the underlying asset (Hull, 2002). Here, the volatility of $\sum_{\theta \in \Theta} \min\left\{ R_{\theta} + C, (1 + \bar{r}_b) B \right\}$ is determined by $\bar{\theta}$: the higher it is, the more volatile the returns among potential borrowers (by the assumptions of SW). Given that $\frac{\partial \theta}{\partial \bar{r}_b} > 0$, it follows that increasing $\bar{r}_b$ raises the volatility of returns, and thereby increases the option value.

For any $\bar{r}_b < \bar{r}_b$ the bank’s expected profit is thus unambiguously increasing in $\bar{r}_b$ (because
both the term in (6) and the option value rise). Hence, \((\tilde{\tau}'')^* < \tilde{\tau}^b\) is impossible, and \((\tilde{\tau}'')^* \geq \tilde{\tau}^b\).

For any \(\tilde{\tau}^b \geq \tilde{\tau}^b\) the trade-off of increasing \(\tilde{\tau}^b\) consists of \(E[\max\{y, k\}]\) being decreased through a lower expected value of \(y\), but being increased by a greater volatility of \(y\).

We next relate this trade-off to capital requirements. The higher is the strike price, \(k\), of a call option, the more valuable is the volatility of \(y\) (Hull, 2002). To see this, consider that for a given value of the underlying asset, a call option with a very low strike price is deep in-the-money and virtually certain to be exercised. Holding such an option becomes virtually equivalent to owning a stock and there is no value to its variance. Instead, an option with a very high strike price is far out-of-the-money, and would not be exercised at a current value of the underlying asset. Its entire value is derived from the variance of that asset, and the probability that before the exercise date the asset value will cross the strike price. The general principle is that volatility is more valuable when the strike price is higher (ceteris paribus). As here \(k = (1 + \tilde{\tau}^d)D\), the volatility of \(y\) is more valuable when \(D\) is higher. And therefore,

\[
\frac{\partial (\tilde{\tau}^b)^*}{\partial D_{\text{max}}} > 0
\]  

(20)

Relating this to credit rationing:

\[
\frac{\partial \Omega}{\partial (Q/X)} > 0 \Leftrightarrow \frac{\partial \Omega}{\partial D_{\text{max}}} < 0
\]  

(21)

\[
\frac{\partial \Omega}{\partial D_{\text{max}}} = \frac{\partial B\left(N - \hat{\theta}\right)}{\partial D_{\text{max}}} - \frac{\partial M X}{\partial D_{\text{max}}} = \frac{\partial B\left(N - \hat{\theta}\right)}{d(\tilde{\tau}^b)^*} \frac{\partial (\tilde{\tau}^b)^*}{\partial D_{\text{max}}} - M \frac{\partial X}{\partial D_{\text{max}}}
\]  

(22)

where \(\frac{\partial B\left(N - \hat{\theta}\right)}{\partial \theta} < 0\) and \(\frac{d\hat{\theta}}{d(\tilde{\tau}^b)} > 0\). Therefore, by equation (20) it follows that \(\frac{\partial B\left(N - \hat{\theta}\right)}{\partial D_{\text{max}}} < 0\). Moreover, \(\frac{\partial X}{\partial D_{\text{max}}} > 0\), so that \(\frac{\partial \Omega}{\partial D_{\text{max}}} < 0\) and higher capital requirements lead to more credit rationing to borrowers. ■
Proof of Lemma 2. By equation (20) in the proof of Lemma 1, we have that $\frac{\partial (\tilde{p}_b)^*}{\partial D} > 0$. In the bank’s maximization problem (equations (8) and (9)) $\tilde{r}_d$ enters through $(1 + \tilde{r}_d) D$. Thus, the sign of $\frac{\partial (\tilde{p}_b)^*}{\partial D}$ must equal that of $\frac{\partial (\tilde{p}_b)^*}{\partial D}$. □

Proof of Lemma 3. Since in (13) $\tilde{r}_d$ adjusts such that the value of

$$E \left[ \min \left\{ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}_b) B \} \right\} - D, \tilde{r}_d D \right]$$

(23)

is kept constant (at zero), it follows that whatever decreases the expected value of

$$\left[ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}_b) B \} \right] - D$$

(24)

induces an increase in $(\tilde{r}_d)^M$.

From equations (8) and (9) the bank solves

$$\max_{\tilde{r}_b} E \left[ \max \left\{ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}_b) B \} \right\} - (1 + \tilde{r}_d) D, 0 \right]$$

(25)

so that its loan rate will always exceed the solution to

$$\max_{\tilde{r}_b} E \left[ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}_b) B \} \right] - D$$

(26)

due to the limited liability option value (see the proof of Lemma 1: $(\tilde{r}_b)^* \geq \tilde{r}_b$).

Thus, any increase in $\tilde{r}_b$ reduces the term in (24), thereby causing an increase in $(\tilde{r}_d)^M$. □

Proof of Proposition 1. When $\frac{\partial (\tilde{p}_d)^M}{\partial \tilde{r}_b} > 0$ then a rise in $D$ (through a change in $D^{\text{max}}$) by increasing $\tilde{r}_b$ (equation (20)) also raises $\tilde{r}_d$. This, in turn, is equivalent to a further increase in $D$: in the bank’s profit function in equation (8) a higher $\tilde{r}_d$ is equivalent to a higher $D$, as both increase $(1 + \tilde{r}_d) D$ (as in the proof of Lemma 2). Therefore, any change in capital requirements gets reinforced when $\frac{\partial (\tilde{p}_d)^*}{\partial \tilde{r}_b} > 0$ rather than $\frac{\partial (\tilde{p}_d)^*}{\partial \tilde{r}_b} = 0$. Hence, $\Omega$ is more sensitive to capital.
requirements when depositors are uninsured:

\[
\frac{\partial \Omega}{\partial (Q/X)} \bigg|_{\frac{\partial (\tilde{r}^d)_{\tilde{r}^b}}{\partial \theta} > 0} > \frac{\partial \Omega}{\partial (Q/X)} \bigg|_{\frac{\partial (\tilde{r}^d)_{\tilde{r}^b}}{\partial \theta} = 0}
\]  

(27)

\[\text{Proof of Lemma 4.} \] This follows the same line of reasoning as the proof of Lemma 3: an increase in \(\Gamma\) causes a rise in

\[
\sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}^b) B \} - D + \Gamma
\]  

(28)

so that, in order to keep

\[
E \left[ \min \left\{ \left[ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}^b) B \} - D + \Gamma, \tilde{r}^d D \right] \right\} \right]
\]  

(29)

equal to zero (equation (14)), it must be that \(\tilde{r}^d\) falls. \(\blacksquare\)

\[\text{Proof of Lemma 5.} \] Rewrite equation (15) to

\[
\max_{\tilde{r}^b} E \left[ \max \left\{ \left[ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}^b) B \} - \left(1 + \tilde{r}^d\right) D + \Gamma, 0 \right] \right\} \right] - \Gamma
\]  

(30)

Here it can be seen that an increase in \(\Gamma\) affects the optimization qualitatively in the same manner as a decrease in \(D\). Hence, by the proof of Lemma 1 (equation (20)), \(\frac{\partial \tilde{r}^b}{\partial \tilde{r}^d} < 0\). \(\blacksquare\)

\[\text{Proof of Proposition 2.} \] First, from Lemma 5, for a given \(\tilde{r}^d\), it holds that \(\frac{\partial \tilde{r}^b}{\partial \tilde{r}^d} < 0\), and by \(\frac{\partial \tilde{r}}{\partial (\tilde{r}^d)_{\tilde{r}^b}} > 0\) and \(\frac{\partial \Omega}{\partial \theta} < 0\) this means that \(\frac{\partial \Omega}{\partial \theta} > 0\). Secondly, the change in \(\tilde{r}^d\) further reinforces this, by Lemma 4’s \(\frac{\partial \tilde{r}^d}{\partial \tilde{r}^b} < 0\) in conjunction with Lemma 2’s \(\frac{\partial (\tilde{r}^b)_{\tilde{r}^d}}{\partial \tilde{r}^d} > 0\). \(\blacksquare\)

\[\text{Proof of Proposition 3.} \] Since

\[
E \left[ \min \left\{ \left[ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \tilde{r}^b) B \} - D, \tilde{r}^d D \right] \right\} \right] \leq \tilde{r}^d D
\]
it follows that

$$\alpha \hat{\tau}^d D + (1 - \alpha) E \left[ \min \left\{ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \hat{\tau}^b) B \} - D, \hat{\tau}^d D \right\} \right]$$

is increasing in $\alpha$, and thus from equation (16) $\frac{\partial (\hat{\tau}^d)^M}{\partial \alpha} < 0$ (following the same argument as in the proofs of Lemma’s 3 and 4).

Hence, by Lemma 2, $\frac{\partial (\hat{\tau}^d)^*}{\partial \alpha} < 0$, which implies, by the proof of Lemma 1, $\frac{\partial \Omega}{\partial \alpha} > 0$. ■

**Proof of Proposition 4.** From equation (16) $\frac{\partial^2 (\hat{\tau}^d)^M}{\partial \phi^2 \partial \alpha} < 0$, as $\lim_{\alpha \to 1} \frac{\partial (\hat{\tau}^d)^M}{\partial \phi} = 0$. It then follows by Proposition 1 that $\frac{\partial^2 \Omega}{\partial (\hat{Q}/X) \partial \alpha} < 0$. ■

**Proof of Proposition 5.** For a fraction $\alpha$ of its funding the bank pays the funding rate that solves

$$(\hat{\tau}^d)^M = \hat{\tau}^d : 0 = \hat{\tau}^d D$$

while for a fraction $(1 - \alpha)$ it is

$$(\hat{\tau}^d)^M = \hat{\tau}^d : 0 = E \left[ \min \left\{ \sum_{\theta \in \Theta} \min \{ R_\theta + C, (1 + \hat{\tau}^b) B \} - D, \hat{\tau}^d D \right\} \right]$$

Taken together this means that the average funding rate that the bank pays is equivalent to equation (16). Hence, $(\hat{\tau}^b)^*$ is unchanged compared to the representative mixed financier, as (by equation (10)) is $\Omega$. ■
References


