Credit Rationing When Banks are Funding Constrained

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Abstract

Credit crunches, such as in the recent financial crisis, generally occur when banks are themselves funding constrained. We use this observation to repair the workhorse Stiglitz-Weiss model of credit rationing. Recent research has invalidated the distributional assumption on which that model is based. This paper shows that by adding the assumption that banks are capacity constrained, Stiglitz-Weiss rationing can occur again. It discusses how this finding can be related to the current policy debates on bank funding and credit provision.

Keywords: Credit rationing, Bank competition, Market structure, Bank funding

JEL Classification: D82, G21

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1 Introduction

Though theories of credit rationing generally model constrained lending by banks as an equilibrium phenomenon that is independent of cycles, empirically rationing occurs primarily at times of severe financial stress, when banks experience difficulty in financing themselves. This was the case during the recent financial crisis, as well as in earlier episodes of credit crunches.\footnote{Empirical work which relates bank funding to the availability of credit include Bernanke et al. (1991), Hancock et al. (1995), Peek and Rosengren (1995a,b), Kashyap and Stein (2000), Curry et al. (2008), Ciccarelli et al. (2010) and Jiménez et al. (forthcoming). For the related theoretical arguments see VanHoose (2007).} In the recent crisis the drying up the wholesale market for bank funding, which grew at a large pace in the preceding years, was particularly poignant. Whether or not banks are funding constrained matters for the way that credit rationing is modelled.

In particular, the model of Stiglitz and Weiss (1981) (henceforth SW) has been the cornerstone of analytical thinking about credit rationing for nearly three decades.\footnote{SW is not the only model of credit rationing. But it is certainly the best known and most often applied model. See Jaffee and Stiglitz (1990) for a discussion of the credit rationing literature.} But a recent paper by Arnold and Riley (2009) (henceforth AR) shows that the key distributional assumption on which the SW-model is based cannot hold, which leaves the economics profession without access to its most-used workhorse for credit rationing, at a time when analysis of rationing is policy relevant. In this paper we show that when banks are funding constrained SW’s result holds again. To understand why this is the case, let us briefly outline the mechanisms of SW’s model and AR’s critique.

In SW’s model credit rationing arises from adverse selection. Borrowers have equal mean returns on their projects, but their projects vary in their riskiness (variance). Because of their limited liability, higher risk borrowers then have larger expected returns, and are willing to pay higher interest rates. Borrowers know the characteristics of their own projects, but they are indistinguishable to banks. Thus banks face a sorting effect when setting their interest rates. Higher rates imply that higher risk projects remain among the pool of borrowers willing to take out a loan. Hence, SW argue, a bank’s expected profits can be hump-shaped.
in interest rates. Initially higher rates raise expected returns. But beyond a threshold the negative sorting effect dominates and higher rates lower expected returns. If the profit-maximizing interest rate is lower than the market clearing rate, there can then be excess demand for loans in equilibrium. This is what SW call rationing: "Among loan applicants who appear to be identical some receive a loan and others do not, and the rejected applicants would not receive a loan even if they offered to pay a higher interest rate" (SW, pp. 394-395).

However, AR show that the hump-shape is untenable. They prove that the expected return per loan always reaches its maximum when only the highest risk borrower remains on the market. The reason is that when a bank sets an interest rate equal to the expected return of this borrower, it appropriates his entire expected return. No loan could earn a higher expected return. Though adverse selection can make the expected return non-monotonic in the loan rate, it cannot be globally hump-shaped. That is, a local "hump" cannot be the global maximum. This is depicted in the top panel of figure 1.

Although SW do not define an explicit bank maximization problem, they implicitly assume a Bertrand competition game (Yanelle, 1989; Freixas and Rochet, 1997, p. 140). What we show is that when banks are funding constrained, the nature of the game changes. Namely, banks compete in capacity constrained Bertrand competition. In many ways this is actually a closer approximation of real world bank competition, because it allows for market power considerations to matter in equilibrium and, empirically, banks do have market power (Bikker and Haaf, 2002; Claessens and Laeven, 2004).

To give the intuition, figure 1 portrays a capacity constrained monopolist bank. The amount lent (panel 2) is kinked: the bank cannot lend more than a given amount. At the right-hand endpoint the bank earns most per loan (panel 1), but lends almost nothing (panel 2). Therefore, its profit function can have an interior global maximum (panel 3). If the interest rate which maximizes expected profit \( r_1 \) in this example) occurs at a point where the demand for loans exceeds what the bank can supply, rationing occurs. This intuition extends to a capacity constrained oligopoly.
The above example also serves to highlight the importance of asymmetric information. Without adverse selection, the expected return per loan in the top panel would be monotonically increasing in the loan rate. The profit maximizing loan rate could then never be at a point where credit is constrained: the bank could always do better by raising the loan rate, grossing a higher return per loan, without losing out on loan demand.

After deriving the main result under completely fixed bank funding, we extend the model to a two-stage game. In the first stage banks can choose how much deposit funding to take on, facing an upward sloping (imperfectly elastic) deposit supply curve. And in the second stage they engage in Bertrand competition, under the capacity constraints determined in the first stage. We solve the game and show that rationing equilibria are still sustainable. This provides a robustness check, in the sense that the result is not driven by an overly rigid assumption on bank funding.

The paper’s outline is as follows. The next section presents the model and its main result. Section 3 extends to endogenous bank capacity. And the concluding section discusses the implications and limitations of the model from a policy perspective.

2 Model

In their paper, SW investigate various modelling setups. We here take their baseline model, in which there is a continuum of borrowers, of mass 1.

A Borrowers

Borrowers are indexed by $\theta \in [0, 1]$, and the return that a borrower makes on his project is called $R_\theta$. The borrower’s projects differ in riskiness. In particular, following SW, it is known that all projects have the same mean return, but they differ in the volatility of returns. The
distributions of projects’ returns are called \( F(R_\theta) \). Importantly, these are private knowledge: a borrower knows the distribution of his project returns, but his bank does not. Borrowers’ projects are sorted by their risk, in the sense that a larger \( \theta \) corresponds to a mean-preserving spread over a smaller \( \theta \). By the definition of second-order stochastic dominance, for \( b > a \):

\[
\int_0^\infty R_b f(R_b) dR = \int_0^\infty R_a f(R_a) dR
\]

and for \( y \geq 0 \)

\[
\int_0^y F(R_b) dR \geq \int_0^y F(R_a) dR
\]

To start up a project, a borrower needs an amount \( B \). If he manages to obtain a loan from a bank he pays the market loan rate, \( r \), for it. This market loan rate is derived below from the equilibrium of the game between banks. If the borrower’s return is insufficient to pay the bank back the promised amount, he defaults on his loan. Formally, this occurs when:

\[
C + R_\theta \leq (1 + r) B
\]

Here \( C \) is the collateral pledged on the loan. The net return to a borrower can be written as:

\[
\max \{R_\theta - (1 + r) B, -C\}
\]

Following SW we define a critical value \( \hat{\theta} \), which means that a borrower applies for a loan from a bank if and only if \( \theta \geq \hat{\theta} \). Any borrower with \( \theta < \hat{\theta} \) chooses not apply for the loan. The reason is that when high returns realize, borrowers make large profits, while with low returns they default. Therefore, greater volatility implies greater expected returns, and a willingness to pay higher loan rates. This, in turn, implies that \( \frac{\partial \hat{\theta}}{\partial r} > 0 \), as shown formally in SW: a higher loan rate brings about a riskier pool of loan applicants. The higher the loan rate, the greater the required expected return for a borrower to find it worthwhile to take out funds and invest in his project. Since, due to limited liability, the riskier borrowers have
greater expected returns, only they remain among the pool of applicants when loan rates rise.

B Banks

On each borrower to which the bank lends it receives

$$\min \{ R_o + C, (1 + r) B \} \quad (5)$$

However, there is a limit to how many loans a bank can issue. Each bank is funded by a fixed amount of retail deposits worth $D$. A bank cannot raise additional deposits, nor does it have any other means of funding. We assume that these deposits pay out an exogenously fixed rate, which for simplicity we set to zero. This assumption is relaxed in the next section.

We assume that there is finite number of banks, which is of importance for the type of competition we define among them below. Each bank can set its own loan rate $r_i$ and, since borrowers are free to apply at any bank they wish, banks thus engage in Bertrand price competition, though subject to capacity constraints (namely, its constrained funding). We solve the game by considering a bank’s decision on $r_i$ for a given rate of all other banks, $r$. The equilibrium occurs when $r_i = r$. That is, the market rate is the loan rate for which no bank has an incentive to deviate.

Before we can turn to solving the game, however, we need to identify the bank’s profit function. Since a bank has no costs, its expected profit is simply the expected return per loan times the number of loans that it makes. The expected return per loan is defined in a manner that is identical to that of AR. Recall that our aim is to show that, in spite of the expected return per loan having the properties found by AR, SW type rationing can occur when we consider banks that are funding constrained. That is why we need the function for the expected return per loan to have the same properties as in AR. We first describe these verbally, and then formally:
Property 1 (Verbal): When the loan rate is zero the expected return per loan is zero.

Property 2 (Verbal): The expected return per loan has an interior local maximum.

Property 3 (Verbal): But it has a global maximum at its right-hand end-point.

These properties describe the picture depicted in the top panel of figure 1 in the introduction. The line starts at the origin, then reaches a local maximum at r1, but has its global maximum at the right-hand end-point of the figure, namely at the loan rate for which only the highest risk borrower is still willing to borrow. Here, SW’s adverse selection mechanism causes the existence of the local maximum in the expected return per loan, but at the loan rate for which only the riskiest borrower ($\theta = 1$) will borrow, the expected return per loan is always larger.

Formally, then, we define $V(r)$ as the expected return per loan and assign it the following properties:

Property 1 (Formal): $V(0) = 0$

Property 2 (Formal): $\lim_{r \to 0^+} V'(r) > 0$

Property 3 (Formal): $\exists r : V'(r) < 0$

Property 4 (Formal): $\arg \max V(r) = r : (\hat{\theta} = 1)$

Here the first formal property is the same as the first verbal property. And the second and third formal properties are a mathematical description of the existence of an interior maximum, which is the second verbal property. Finally, the fourth formal property represents the third verbal one.

Proposition 1 There exists a $\tilde{D}$ such that when $D < \tilde{D}$ rationing as defined by SW is sustainable as an equilibrium outcome.
Proof. In the appendix. 

Proposition 1 is our main result. It shows that, in spite of the distributional considerations introduced by AR, credit rationing is a feasible equilibrium. The reason is the capacity constraints of banks, and how this translates into market power. After all, a given bank will only find it profitable to deviate from the rationing equilibrium if its profits after deviation exceed those it obtains at the equilibrium. When there is unconstrained Bertrand competition, some bank will always deviate because it can capture the entire market share by doing so. However, when funding constraints play a role, banks see more limited gains to deviation. Within the rationing equilibrium, each bank has positive profits, and potentially little to gain from undercutting. This is what makes that equilibrium sustainable.

3 Endogenous capacity

In the basic model banks were given fixed deposits \( D \). In this section we make this assumption less rigid by considering a two-stage game, in which the first stage consists of each bank choosing the amount of deposits that it wishes to take on, \( D_i \), while in the second stage banks compete in the same manner as in the previous section. In the first stage, when turning to depositors, banks face an imperfectly elastic supply of deposits. That is, the more deposits banks attract, the higher the deposit rate that they have to pay on the market. Here we call that deposit rate \( \rho \), and we write it as a function of how much deposits banks attract overall, \( \sum_j D_j \). In particular, \( \rho' \left( \sum_j D_j \right) > 0 \) represents the upward-sloping supply curve. A bank’s profits are now:

\[
V(r_i)D_i - \rho \left( \sum_j D_j \right) D_i
\]

Here \( r_i \) are determined at the second stage of the game, when banks compete for borrowers. At that stage capacities \( (D_i) \) are fixed, and thus the second stage game proceeds just like in the previous section. However, at the first stage banks determine \( D_i \) taking into account
how the second stage will be affected.

**Proposition 2** There exist parameterizations such that rationing as defined by SW is a subgame perfect Nash equilibrium.

**Proof.** In the appendix.

Proposition 2 shows that in spite of banks’ ability to expand capacity in the first stage, they do not always want to take on enough deposits to eliminate rationing at the second stage. Although the trade-off behind increasing loan supply is now more "smooth" than it was in the previous section, its basic intuition is unchanged. That is, banks are now allowed to raise more funds, but at an increasing cost. A strategy of deviation from a rationing equilibrium can still prove unprofitable. Local deviation, i.e. near the rationing equilibrium, is not a rewarding strategy because the expected return per loan has a local maximum there, like in figure 1’s top panel. And a large deviation, whereby a bank significantly cuts loan rates, while taking on a lot more funds and expanding market share, may or may not be worthwhile, depending on how the cost of funding reacts.

### 4 Policy implications

The financial crisis has put the possibility of borrower rationing back into the minds of policymakers. Firstly because during the crisis funding constraints among banks rapidly translated into a credit crunch. And secondly because in the implementation of the transition to new capital regulation, a key issue is to ensure that banks are not forced to delever too quickly as this may result in rationed bank borrowers (BIS, 2010). With such issues in mind it is important to possess a functioning analytical tool for credit rationing. The aim of this paper has been to overcome the technical issues that invalidated the workhorse Stiglitz-Weiss model. It has done so by relying on the funding side constraints that have proven pivotal to banks in recent years.
Discussing policy implications in light of the SW model comes with several important limitations, however. Most crucially perhaps, SW is a microeconomic model, and cannot be easily translated to macroeconomic implications. Whether a decline in bank funding translates into an economy-wide credit crunch depends on whether borrowers are able to access other forms of finance, after all (Werner, 2005, 2011). As pointed out by Werner (2011, pp. 26-27):

“The credit view consists of the bank lending and balance sheet approaches, and the credit rationing argument (Jaffee and Russell, 1976; Stiglitz and Weiss, 1981). The latter is undoubtedly correct: it maintains that banks keep interest rates below the hypothetical equilibrium level, since demand for money and credit is always large, then ration and allocate credit, to minimize non-performing loans. But it is a microeconomic theory in search of macroeconomic consequences: lack of bank credit, we are told, can be easily compensated by credit from non-bank financial intermediaries or ‘direct finance’ in the form of public debt, or equity. However, empirically this does not happen. When banks are damaged, go bust, are closed or even restructured, the economy tends to suffer (Peek and Rosengren, 2000; Ashcraft, 2005; Leary, 2009; Voutsinas and Werner, 2011).”

In fact, substitutability of bank funding differs across countries, with EU countries being more dependent, in particular among SME’s, than the US. Although some attempts have been made to take SW closer to the macro level (Suarez and Sussman, 1997), the empirical macro literature still tends to regard credit crunches simply as shifts in credit supply (Bernanke et al., 1991) rather than rationing episodes. The welfare effects of reduced credit supply within a standard price-mechanism and rationing could be quite different, however.³

Having said so, the key point of the paper, that bank funding constraints can translate into borrower credit constraints, would seem to be broader than the particular issues surrounding the SW model. If we may venture a bit beyond its confines, one reason that

³For the price adjustment mechanism see Hawtrey and Liang (2008) and references therein.
financial intermediation takes place through banks is that banks are thought to have better knowledge of borrowers. Thus, if banks suffer from asymmetric information, this would only be worse for any other form of funding. Hence the same rationing incentives that apply to banks would apply to credit provision in the economy more generally. A similar thing can be said of the availability of funding. In some sense banks are more likely than other agents to be able to obtain funding for their loans, not in the least because they are protected by explicit or implicit bailout guarantees. At a time that banks find themselves funding constrained, non-bank intermediaries or direct financiers would find themselves in a similar position. If so, then the findings of this paper may be applicable at a macro level too.

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Appendix: Proofs

Proof of Proposition 1. First we show that the $r$ for which $V'(r) = 0$ is the only candidate equilibrium for rationing as defined by SW (pp. 394-395). Then we show that $\exists \tilde{D}$ such that for $D < \tilde{D}$ no bank will want deviate from this $r$, and, therefore, it is a sustainable equilibrium.

Define $r^* = r : (V'(r) = 0)$. Rationing as defined by SW (pp. 394-395) comes about when at the equilibrium market loan rate loan demand exceeds loan supply. Loan supply is $(M)(D)$ whereas loan demand is

$$\int_{\hat{\theta}}^{1} B d\theta = B \left(1 - \hat{\theta}\right)$$

so that rationing occurs when at $r$

$$B \left(1 - \hat{\theta}\right) > MD$$

$\Leftrightarrow$

$$\hat{\theta} < \frac{B - MD}{B}$$

(8)

First assume that this condition is satisfied at $r^*$. Then, by $\frac{d\hat{\theta}}{dr} > 0$, it is also satisfied at any $r < r^*$. This means that if $r < r^*$ a bank could raise $r_i$ without losing borrowers, because other banks could not meet the additional demand. However, by raising $r_i$ it also increases $V(r_i)$ by Formal Properties 2 and 3. Therefore, its profits rise, and any $r < r^*$ could not be a sustainable equilibrium with rationing. But if there exist values of $r > r^*$ for which condition (8) holds, these could also not be sustainable. A bank would then raise $V(r_i)$ by lowering $r_i$ below $r$ while similarly losing no borrowers. Therefore, $r^*$ is the only candidate rationing equilibrium.
It remains to show that this equilibrium can indeed come about. Firstly, as the right hand side of (8) decreases in $D$ it follows that for any mapping of $r$ to $\hat{\theta}$ there exists a $\bar{D}$ such that for $D < \bar{D}$ rationing occurs at $r^*$. That no local deviation of any $r_i$ from $r^*$ is optimal, already follows from the arguments in the previous paragraph: such a change lowers $V(r_i)$ without raising the amount lent. It thus remains to show that there are no global deviations of $r_i$ that are profitable. By Formal Property 4 there exist values of $r_i > r^*$ for which $V(r_i) > V(r^*)$. However, by $\frac{\partial \hat{\theta}}{\partial r} > 0$ it follows that loan demand, $B \left(1 - \hat{\theta}\right)$, falls, as does expected profit. Therefore, there exist functions $V(r)$ that satisfy Formal Properties 1-4, and for which $r^*$ is the unique equilibrium loan rate. 

**Proof of Proposition 2.** Suppose not. Then at the stage-two equilibrium loan rate $r^*$ it could never hold that

$$B \left(1 - \hat{\theta}\right) > \sum_j D_j$$

At loan rate $r^*$ bank profit is

$$\left[V(r^*) - \rho \left(\sum_j D_j\right)\right] D_i$$

which by $\rho' \left(\sum_j D_j\right) > 0$ has a global maximum in $D_i$. Thus, there exists a $D_i^*$. Functions $V(r)$ and $\rho \left(\sum_j D_j\right) > 0$ can be arbitrarily chosen such that

$$B \left(1 - \hat{\theta}(r^*)\right) > MD_i^*$$

which proves the contradiction. 

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References


